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Corrugation-Type Instability of the Free Motion of 180° Domain Walls in Uniaxial Ferromagnets

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Abstract—Investigation of the spectrum of oscillations of a 180° domain wall (DW) freely moving in a uniaxial ferromagnet shows that the one-dimensional structure becomes unstable with respect to surface distortions in the region of negative effective mass (negative differential mobility). The upper critical value of the perturbation wave vector, above which the DW corrugation is not developed, and the wave vector of a perturbation mode with the maximum increment are determined. © *2003 MAIK "Nauka/Interperiodica".*

Development of the information technologies accounts for the interest in investigations of the soliton dynamics in magnetic media. The simplest one-dimensional (1D) object of this kind in uniaxial ferromagnets is a kink representing a 180° domain wall (DW). On the whole, the dynamics of such DWs is rather exhaustively studied (see the Walker solution of 1956 for a DW in an external magnetic field [1] and the Schloemann solution for freely moving DWs [2, 3]). The stability of solutions of the nonlinear Landau–Lifshits equations with respect to development of a corrugation type instability (transverse bending of the DW surface) has been studied in less detail, although such corrugated DWs are rather frequently observed in experiment. Examples, even restricted to the case of so-called films with a transverse uniaxial magnetic anisotropy, are offered (see [4]) by static distortions of the circular cross section of magnetic bubbles, bending distortions of a flat DW according to Schloemann, dynamic gyrotropic inflection of a DW surface under the action of a moving Bloch line, etc. It should be noted that, judging by these and other experimental data (for flat DWs, see, e.g., [5, 6]), there is no common mechanism responsible for the DW corrugation.

This study addresses the simplest case of corrugation arising on the surface of a one-dimensional 180° DW freely moving in a uniaxial ferromagnet. In other words, we will consider the instability of the Walker solution in the Schloemann form [2, 3]. Doubts concerning the stability of this solution are related to a region with a negative differential mobility of DWs (in terms of [1]) of with a negative effective mass (in terms of [2, 3]). According to the qualitative arguments [7], the 1D motion of DWs in such regions can be unstable with respect to corrugation. This problem was specially studied on a spectral level [8, 9] and it was demonstrated [9] that DWs actually exhibit a corrugation-type instability in the region of negative mobility. Unfortunately, the spectrum presented in [9] is restricted to the case of a linear spatial dispersion with respect to the 2D perturbation wave vector $\mathbf{k}_{\parallel}$ localized on the DW surface.

In this study, the spectrum of localized oscillations is derived from the spectral equations of stability under not as strong limitations on $\mathbf{k}_{\parallel}$ as in [9]. This approach will allow us determine the wavenumber $k_{\parallel M}$ corresponding to a mode with the maximum increment in the region of instability and, hence, determining the period of corrugation. In addition, it will be demonstrated that there is another critical value of $k_{\parallel} = k_{\parallel b}$ ($>k_{\parallel M}$), above which the DWs remain stable with respect to corrugation.

Consider a 180° DW in the $x0z$ plane, freely moving in a uniaxial ferromagnet in the positive direction of the $0y$ axis (the easy axis is assumed to be collinear with the $0z$ axis). The corresponding 1D problem solution is well known [2, 3] and can be written as

$$\begin{aligned} \sin\theta_0(y) &= 1/\cosh[(y - Vt)/\Delta(\varphi_0)], \\ \Delta(\varphi_0) &= (1 + \sin^2\varphi_0/Q)^{-1/2}, \\ \sin^2\varphi_M &= Q((1 + 1/Q)^{1/2} - 1), \\ V &= \Delta(\varphi_0)\sin\varphi_0\cos\varphi_0/Q. \end{aligned} \tag{1}$$

Here $\theta_0(y)$ and φ_0 = const are the polar and azimuthal angles of the magnetization vector $\mathbf{M}$, respectively, measured from the $0z$ and $0x$ axes (lying in the DW plane); the y coordinate is measured in units of the Bloch DW width $\Delta = (A/K)^{1/2}$, where A is the exchange hardness and K is the uniaxial anisotropy constant; the time is measured in the units of $(\gamma H_a)^{-1}$, where $H_a = 2K/M$ is the uniaxial anisotropy field; $Q = H_a/4\pi M$ is the so-called quality factor of the material; and $\varphi_M(Q)$ is the value corresponding to the maximum DW veloc-

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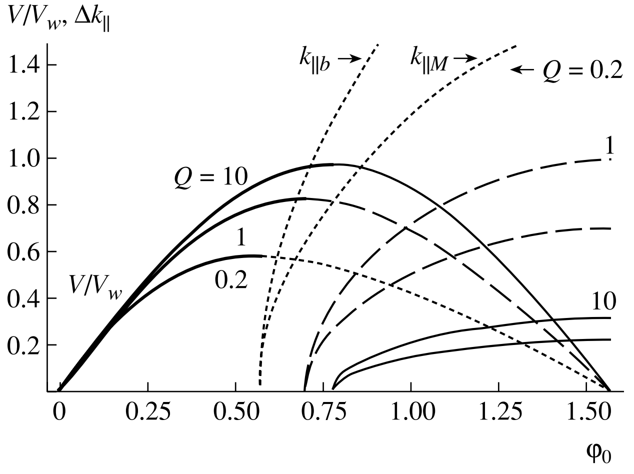

Figure.

ity. Plots of the DW velocity $V(\varphi_0)$ (in the units of $V_w = 2\pi\gamma M\Delta$) for three typical values of Q are presented in the figure, where the regions of stability (see below) are indicated by thick solid lines. The quantity φ_0 has a meaning of the momentum of a DW whose Hamiltonian $H = \Delta(\varphi_0)^{-1}$ is a periodic function of this momentum [10] (in what follows, the consideration can be restricted to the interval $0 < \varphi_0 < \pi/2$). In the descending branches of $V(\varphi_0)$ (see figure), the effective mass m of the DW is negative ($V = \partial H/\partial\varphi_0$ and $1/m = \partial V/\partial\varphi_0 < 0$).

Equations describing the case of small oscillations can be obtained by passing to a local coordinate system moving at the velocity V with the DW along the $0y$ axis. The axes of the new system in the base plane $x0y$ are rotated by the angle φ_0, so that the new $0x$ axis coincides with the plane in which the spins are rotated by 180°. For small amplitudes of magnetization ($\sim\exp(-i\omega t + i\tilde{k}_\parallel \rho)$), where $\rho = (x, z)$ are the coordinates in the DW plane) occurring in the plane of spin rotation ($m_\parallel$) and that perpendicular to this plane ($m_\perp$), we obtain the equations

$$\begin{aligned} i\tilde{\omega}m_\perp + \tilde{V}\hat{L}^+ m_\perp + (\hat{L} + \tilde{k}_\parallel^2)m_\parallel &= 0, \\ -i\tilde{\omega}m_\parallel + \tilde{V}\hat{L}^- m_\parallel + (\hat{L} + \tilde{k}_\parallel^2 + \tilde{\omega}_{ms})m_\perp &= 0, \end{aligned} \qquad (2)$$

which coincide to within the notation with the results obtained in [9, 11]. Here, $\hat{L} = \hat{L}^+\hat{L}^- = -d^2/dy^2 + 1 - 2/\cosh^2 y$ and $\hat{L}^\pm = \pm d/dy - \tanh y$ are operators and the local coordinate y implies $(y - V_t)\Delta(\varphi_0)$; $\tilde{\omega} = \omega\Delta(\varphi_0)^2$, $\tilde{k}_\parallel = k_\parallel\Delta(\varphi_0)$, $\tilde{V} = V\Delta(\varphi_0)$, and $\tilde{\omega}_{ms} = Q^{-1}(\varphi_0)\cos^2\varphi_0$ (see formulas in (1)).

The main difficulty encountered in solving Eqs. (2) is related to taking into account the terms proportional to $\sim\tilde{V}$ (the static case of (2) with $\tilde{V} = 0$ is well known, see Winter (1961)). It would be natural to employ the perturbation theory with respect to $\tilde{V}$, expanding Eqs. (2) in terms of the complete orthonormalized basis set of operator $\hat{L}$. The spectrum of this operator consists of two parts: (i) a translation mode $\chi_{tr}(y) = 1/(\sqrt{2}\cosh y)$, $\hat{L}\chi_{tr}(y) = \hat{L}^-\chi_{tr}(y) = 0$ localized on the DW and (ii) a precession mode $\chi_{pr}(y, k) = \hat{L}^+\exp(iky)/\sqrt{2\pi(1 + k^2)}$, $\hat{L}\chi_{pr}(y, k) = (1 + k^2)\chi_{pr}(y, k)$, localized primarily in the domains. According to Eqs. (2), the corresponding resonance frequencies are (i) $\Omega_{tr}^{(0)}(\tilde{k}_\parallel)^2 = \tilde{k}_\parallel^2(\tilde{k}_\parallel^2 + \tilde{\omega}_{ms})$ (translation level) and (ii) $\Omega_{pr}^{(0)}(\tilde{k}_\parallel, k)^2 = (1 + k^2 + \tilde{k}_\parallel^2)(1 + \tilde{k}_\parallel^2 + k^2 + \tilde{\omega}_{ms}) > \Omega_{pr}^{(0)}(\tilde{k}_\parallel)^2$ (precession level).

The corrugation instability is determined on the translation level and the corresponding frequency corrections to $\Omega_{tr}^{(0)}(\tilde{k}_\parallel)$ have to be determined. The perturbation theory employed here as analogous to the scheme used in [11], where the calculations were restricted to the first order in $\tilde{V}$. Since this approximation corresponds, as demonstrated in [11] and confirmed by our calculations, to the first-order correction $\Omega_{tr}^{(1)}(\tilde{k}_\parallel) = 0$, the refined calculation should include the second-order terms. The contribution of the diagonal term $\tilde{\omega}_{ms}(\varphi_0)$ depending on the DW velocity is taken into account to within the corresponding order of the perturbation theory. The results are as follows:

$$m_{\parallel tr}(y) = \frac{-i\Omega_{tr}^{(0)}(\tilde{k}_\parallel)}{\tilde{k}_\parallel^2}m_{\perp tr}(y) = \chi_{tr}(y) - i\tilde{V}\frac{\tilde{k}_\parallel^2}{\Omega_{tr}^{(0)}(\tilde{k}_\parallel)} \times \int_{-\infty}^{\infty} dk \frac{(1 + k^2 + \tilde{k}_\parallel^2)\langle\chi_{pr}(y_1, k)|\hat{L}^+|\chi_{tr}(y_1)\rangle}{\Omega_{pr}^{(0)}(\tilde{k}_\parallel, k)^2 + \Omega_{tr}^{(0)}(\tilde{k}_\parallel)^2}\chi_{pr}(y, k); \qquad (3.1)$$

$$\Omega_{tr}(\tilde{k}_\parallel)^2 = \tilde{k}_\parallel^2(\tilde{\omega}_{ms} + \tilde{k}_\parallel^2 - \tilde{V}^2 I(\tilde{k}_\parallel)), \qquad (3.2)$$

$$I(\tilde{k}_\parallel) = \int_{-\infty}^{\infty} dk \times \frac{(1 + k^2 + \tilde{k}_\parallel^2 + \tilde{\omega}_{ms})|\langle\chi_{pr}(y_1, k)|\hat{L}^+|\chi_{tr}(y_1)\rangle|^2}{\Omega_{pr}^{(0)}(\tilde{k}_\parallel, k)^2 + \Omega_{tr}^{(0)}(\tilde{k}_\parallel)^2} > 0, \qquad (3.3)$$

where $\langle\ldots\rangle$ denotes the operation of determining the matrix element (integrating with respect to y_1 between infinite limits), so that $|\langle\chi_{pr}(y_1, k)|\hat{L}^+|\chi_{pr}(y_1)\rangle|^2 = (\pi/4)(1 + k^2)/\cosh^2(\pi k/2)$.

In the lowest order with respect to $\tilde{k}_{\|}$, Eqs. (3.2) and (3.3) yield (note that $I(0) = 1$ independently of $\tilde{\omega}_{ms}(\varphi_0)$) an expression

$$\omega_{tr}(\tilde{k}_{\|} \longrightarrow 0)^2 = \tilde{k}_{\|}^2(\tilde{\omega}_{ms} - \tilde{V}^2) = (\tilde{k}_{\|}^2\Delta(\varphi_0)\partial V(\varphi_0)/\partial\varphi_0,) \quad (4)$$

which coincides in the vicinity of the DW velocity maximum (where $\varphi_0 = \varphi_M$) with the results of [9]. This expression shows that, in the region where $\partial V(\varphi_0)/\partial\varphi_0 < 0$ (negative effective mass), the DW becomes unstable with respect to the surface perturbations with $\tilde{k}_{\|} \neq 0$. However, in contrast to [9], the complete expressions (3.2) and (3.3) allow some additional features of the corrugation instability to be determined.

Expansion of the $I(\tilde{k}_{\|})$ integral into series in $\tilde{k}_{\|}$ shows that the terms on the order of $\tilde{k}_{\|}^4$ in (3.2) are always positive. With allowance for (4), this fact indicates that there exists a certain value $k_{\|} = k_{\|M}$ for which the increment of the corrugation instability reaches maximum and, hence, determines the most probable value of the steady-state corrugation period. At the same time, since $\Omega_{tr}(\tilde{k}_{\|} = 0)^2 = 0$ and $\Omega_{tr}(\tilde{k}_{\|} \longrightarrow \infty)^2 = \tilde{k}_{\|}^4$ (for $I(\tilde{k}_{\|} \longrightarrow \infty) \longrightarrow$ const), we may conclude that there is another critical value of $k_{\|} = k_{\|b}$ (besides $k_{\|} = 0$), at which $\Omega_{tr}(\tilde{k}_{\|b})^2 = 0$ and above which $\Omega_{tr}(\tilde{k}_{\|})^2$ is always positive (i.e., the corrugation disappears). The figure presents three pairs of the $k_{\|M}(\varphi_0)$ and $k_{\|b}(\varphi_0)$ curves (numerically calculated for $Q = 1.2$, 1, and 10) originating from the points $\varphi_M(Q)$ on the abscissa axis (for a given Q, $k_{\|b}(\varphi_0)$ is situated above $k_{\|M}(\varphi_0)$). As for the applicability of the perturbation theory, it is known that this requires the first-order correction in Eq. (3.1) to be small. In the case of $Q \gg 1$, this correction can be shown to be always small (~$1/Q$). The case of $Q \ll 1$ is more problematic: except for separate regions, the smallness of the first-order correction depends on the smallness of $k_{\|}$.

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SPELL: 1. dumka or Dumka—?