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MAGNETISM AND FERROELECTRICITY

Translational Motion of Domain Walls in a Strong Magnetic Field Circularly Polarized in the Basal Plane of a Uniaxial Ferromagnet

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Abstract—A substantially nonlinear theory is developed for the translational motion of domain walls (DWs) in ferromagnets with large easy-axis anisotropy under the influence of a strong magnetic field circularly polarized in the basal plane of the ferromagnet. This theory is a generalization of the well-known theories of DW drift that are limited to an approximation quadratic in the field magnitude. The analytical results are confirmed by computer simulation performed on the basis of the Landau–Lifshitz equations.

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1. INTRODUCTION

The dynamics of domain walls (DWs) in ferromagnetic materials under the influence of an ac magnetic field has a number of peculiar features. One of them is DW drift, a nonlinear phenomenon that consists in translational DW motion resulting from averaging over oscillations of the external magnetic field. The simplest situation, in which the DW drift is observed and which is the only situation considered in the present paper, is realized for 180° DWs in uniaxial ferromagnets with easy-axis magnetic anisotropy. In particular, the DW drift appears if a sample is subjected to a magnetic field circularly polarized in the basal plane of the ferromagnet. This field gives rise to an unbalanced magnetic pressure and sets a DW in motion. In a system of domains, the unbalanced magnetic pressure can cause (depending on the field polarization) a somewhat different but closely related effect, the reorientation of DWs in space. A review of these topics covering up to 1979 that mainly concerns the case of ferromagnets in which the uniaxial-anisotropy energy dominates over magnetostatic interactions can be found in [1].

Certainly, DW drift is a significantly nonlinear effect and is at least quadratic in the amplitude of the exciting field. Unfortunately, calculations of the DW drift velocity as a function of the field amplitude performed in the framework of the existing theories [2–5] have been limited to the lowest nonzero (second-order) approximation with respect to the amplitude of the external field. The purpose of the present paper is to develop a substantially nonlinear theory of DW drift going beyond this approximation.

It turns out that this problem can be solved for uniaxial easy-axis ferromagnets if the quality factor Q (the ratio of the magnetic anisotropy energy to the magnetostatic interaction energy) satisfies the condition

$$Q \equiv K/2\pi M^2 \equiv H_a/4\pi M > 1, \quad (1)$$

where K is the uniaxial anisotropy constant, M is the magnetization, and H_a is the effective anisotropy field. In this case, the nonlinear problem of the DW drift can be solved analytically over a wide range of applied external fields H, with the field amplitude being limited from above by the inequality $H < H_a$. The Landau–Lifshitz equations can be reduced to the Slonczewski equations [1, 6], with which the problem reduces to a set of nonlinear ordinary differential equations instead of partial differential equations. A strong circularly polarized magnetic field drags the spins that are localized in the center of the DW and causes the spins to rotate (a weak field causes only small deflections of the spins). The resulting dissipative response leads to the translational DW displacement.

The general theory of DW drift is necessary, if nothing else, because many experimental studies on this subject have been performed in relatively strong magnetic fields and, as a result, the observed effects go beyond the limits of applicability of the approximation quadratic in the field amplitude (see, e.g., [7–9]). This argument is also fully relevant to important topics, such as DW dynamics and electromagnetic losses in soft magnetic materials (with $Q < 1$) in the case where the applied magnetic field (or a sample) rotates [10–13] and the DW drift also occurs.

2. THE PROBLEM AND MAIN EQUATIONS

For a uniaxial easy-axis ferromagnet (with the easy axis assumed to be parallel to the z axis), the initial energy density w and the equations of motions (in the one-dimensional case) can be written as

$$w = \frac{A}{M^2}\mathbf{M}'^2 - \frac{K}{M^2}M_z^2 + 2\pi M_y^2 - \mathbf{HM}, \tag{2a}$$

$$\dot{\mathbf{M}} = \gamma[\mathbf{H}^{\text{eff}}, \mathbf{M}] + \frac{\alpha}{M}[\mathbf{M}, \dot{\mathbf{M}}]. \tag{2b}$$

The expression for w assumes that the magnetization $\mathbf{M}(y, t)$ depends only on the spatial coordinate y and time t. Here and henceforth, the derivatives with respect to these variables are denoted by a prime and a superscript dot, respectively. The dissipative term in the Landau–Lifshitz equations (2b) is presented in the Gilbert form ($\gamma > 0$ is the gyromagnetic ratio, $\alpha > 0$ is the damping constant). On the right-hand side of Eq. (2a), the first term is the nonuniform exchange energy ($A > 0$ is the exchange stiffness), the second term is the crystallographic magnetic anisotropy energy ($K > 0$ is the anisotropy constant, the easy axis is parallel to the z axis), the third term is the magnetostatic energy in a one-dimensional approximation, and the fourth term is the Zeeman energy ($\mathbf{H}$ is the external magnetic field vector). The effective internal field is defined in the usual way as the variational derivative, $\mathbf{H}^{\text{eff}} = -\delta w/\delta\mathbf{M}$. In what follows, we consider a DW located in the xz plane; so the normal to the DW plane is directed along the y axis. At $\mathbf{H} = 0$, the DW has a well-known Bloch structure; so the DW magnetization has no component along the y axis:

$$\mathbf{M}(y-q) = M_x\mathbf{e}_x + M_z\mathbf{e}_z$$
$$\equiv M(\sin\theta(y-q)\mathbf{e}_x + \cos\theta(y-q)\mathbf{e}_z), \tag{3}$$

where θ is the polar angle of the magnetization vector measured from the positive direction of the z axis, q is the coordinate of the center of the DW, $\cos\theta(y-q) = -\tanh[(y-q)/\Delta]$, $d\theta/dy > 0$, and $\Delta = (A/K)^{1/2}$ is the DW width parameter.

As was said above, calculations of the drift velocity based on iterative solutions to Eqs. (2) [2–5] have been limited to terms of the second order in an external (weak) field. This limitation can be removed if we consider ferromagnets with $Q > 1$. In this case, the DW dynamics in a magnetic field can be described in terms of the Slonczewski equations, which include only the coordinate of the DW center $q(t)$ (see Eq. (3)) and the azimuth angle of the magnetization vector $\psi(t)$ at $\theta = \pi/2$, i.e., in the center of the DW. When applied to a 180° DW described by Eq. (3), these equations in the one-dimensional case reduce to the ordinary differential equations

$$\dot{\psi} - H_z + \alpha\dot{q} = 0, \tag{4a}$$

$$\dot{q} - \alpha\dot{\psi} = \sin\psi\cos\psi + H_x\sin\psi - H_y\cos\psi. \tag{4b}$$

In Eqs. (4), the following dimensionless variables are introduced (indicated on the right of the arrows):

$$t \longrightarrow t/(4\pi\gamma M), \quad H_z \longrightarrow 4\pi M H_z,$$
$$H_{x,y} \longrightarrow 8MH_{x,y}, \quad q \longrightarrow \Delta q. \tag{5}$$

A consistent derivation of Eqs. (4) (or of their more complete version) from the Landau–Lifshitz equations using the method of asymptotic expansions [14, 15] demonstrates that effects of the order of $1/Q$ are neglected in Eqs. (4).

For the problem in question, the most important of these neglected effects are the deflection of the vector $\mathbf{M}$ from the easy axis (z) and the excitation of uniform precession of the spins deep in domains under the influence of the field $\mathbf{H}$. Equations (4) are valid and the above effects are small if the amplitude and frequency of the external field satisfy the inequalities

$$H_{x,y} < H_a, \quad \omega < \gamma H_a. \tag{6}$$

3. APPROXIMATE EQUATIONS AND THE DRIFT OF THE DOMAIN WALL UNDER A STRONG ROTATING MAGNETIC FIELD

The problem of DW drift can easily be solved using Eqs. (4). The rotating field $\mathbf{H}$ can be written as $H_x = H\cos\omega t$, $H_y = H\sin\omega t$, and $H_z = 0$. Introducing the phase angle by which the magnetization in the center of the DW lags behind the magnetic field in the basal plane, $\Phi = \psi - \omega t$, Eqs. (4) can be reduced to

$$\dot{q} = -\frac{1}{\alpha}(\dot{\Phi} + \omega), \tag{7a}$$

$$\frac{1+\alpha^2}{\alpha}(\dot{\Phi} + \omega) + H\sin\Phi = -\sin 2(\Phi + \omega t)/2. \tag{7b}$$

In order to solve the set of approximate equations (7), we make the following assumption, which is easier to formulate using Eqs. (4). In the quasi-static limit, the angle ψ can be found by equating the right-hand side of Eq. (4b) to zero. The equation thus obtained, as well as the energy from which this equation can be derived, has absolutely the same structure as the equations arising in the problem of stability of a uniaxial ferromagnet in a magnetic field. The easy directions correspond to the angles $\psi = 0$ and π in the DW plane, and the anisotropy field H_a is replaced by $8M$. On the grounds of this similarity, we can conclude that the magnetic field unambiguously defines the DW magnetization (a stable value of ψ) if the vector $\mathbf{H}$ in the plane (H_x, H_y) falls beyond the astroid 1

$$H \equiv (H_x^2 + H_y^2)^{1/2} > 1, \quad H_x^{2/3} + H_y^{2/3} = 1, \tag{8}$$

where the fields $H_{x,y}$ in the basal plane are measured in units of $8M$. Below, only this case of strong fields is considered. In this case, the right-hand side of Eq. (7b) can be set equal to zero in a first approximation.

In this approximation, the set of equations (7) has two kinds of solutions. In one (low-frequency) solution, the magnetization in the center of the DW rotates synchronously with the external field **H** and lags behind it in phase by a constant angle $\Phi = \Phi_0$. The DW velocity $\dot{q}$, the constant phase Φ_0, and the limits of the occurrence of this mode of DW motion are defined by the expressions

$$\dot{q} = -\omega/\alpha, \quad \sin\Phi_0 = -(1+\alpha^2)\omega/\alpha H, \quad \dot{\Phi}_0 = 0,$$
$$\omega \le \omega_c, \quad \omega_c = \alpha H/(1+\alpha^2), \tag{9}$$
$$|\dot{q}| \le |\dot{q}_c| = H/(1+\alpha^2).$$

This mode is limited from above by the maximum frequency ω_c at which the negative DW velocity reaches a maximum absolute value $V = |\dot{q}_c|$ (the sign of the DW velocity depends, in particular, on the rotation direction of the magnetic field) and the stationary phase (which varies in the range $0 \le \Phi_0 \le -\pi/2$) is equal to $\Phi_0 = -\pi/2$.

Solutions of the other kind occur at frequencies $\omega > \omega_c$. Here, the phase angle between the magnetization in the center of the DW (which follows the rotating magnetic field as before) and the field oscillates and the lag of the magnetization direction behind that of the field increases with frequency:

$$\sin\Phi(t) = -\frac{\omega_c/\omega + \sin(\Omega t)}{1 + \omega_c/\omega\sin(\Omega t)},$$
$$\dot{\Phi}(t) = \frac{-(\omega^2 - \omega_c^2)/\omega}{1 + \omega_c/\omega\sin(\Omega t)}, \tag{10}$$

where ω_c is given by Eq. (9) and $\Omega = (\omega^2 - \omega_c^2)^{1/2}$. Let us average Eqs. (10) over the oscillation period $T = 2\pi/\Omega$. By directly integrating Eqs. (10), we find the average values

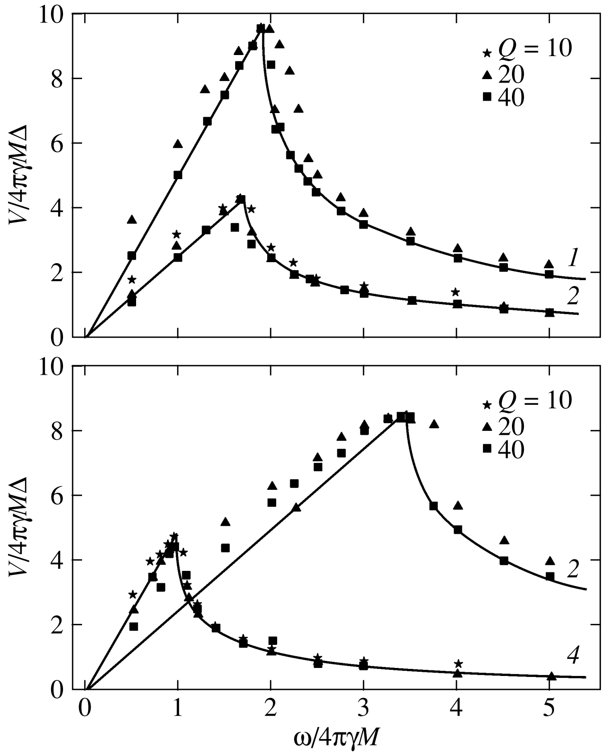
$$\overline{\sin\Phi} = -\omega_c/(\omega + (\omega^2 - \omega_c^2)^{1/2}),$$
$$\bar{\dot{\Phi}} = -(\omega^2 - \omega_c^2)^{1/2}. \tag{11}$$

Substituting $\bar{\dot{\Phi}}$ into Eq. (7a), we find the mean DW velocity

$$\bar{\dot{q}} = -(\omega - \sqrt{\omega^2 - \omega_c^2})/\alpha. \tag{12}$$

This expression is valid in the range $\omega > \omega_c \equiv \alpha H/(1 + \alpha^2)$ [see Eq. (9)]. At the point $\omega = \omega_c$, Eqs. (12) and (9) coincide. As the frequency grows, the mean DW velocity (12) decreases steadily and tends to zero in the range $Q > \omega > 1$ as

$$\bar{\dot{q}} \longrightarrow -\frac{\alpha H^2}{2(1+\alpha^2)\omega}. \tag{13}$$



Mean DW velocity $V \equiv |\dot{q}|$ as a function of the frequency ω of the magnetic field H rotating in the basal plane of the ferromagnet calculated for various values of the Gilbert damping constant α and quality factor Q. The solid lines are calculations based on the Slonczewski equations, and symbols are the results of numerical integration of the Landau–Lifshitz equations for several values of Q: (*1*) $H = 10$ and $\alpha = 0.2$; (*2*) $H = 5$ and $\alpha = 0.4$; (*3*) $H = 10$ and $\alpha = 0.4$; and (*4*) $H = 5$ and $\alpha = 0.2$.

In Eq. (13), the frequency is limited from above by the inequality $\omega < Q$, which can be seen from the second of inequalities (6) if we express this inequality in terms of dimensionless variables (5). On the whole, the frequency dependence of the DW drift velocity is described by Eqs. (9) and (12), as shown by the solid lines in the figure (V is the magnitude of the velocity). The peak observed at the boundary point between the two different modes of DW motion $\omega = \omega_c$ [see Eq.(9)] and the constancy of the sign of the DW velocity are typical features of this dependence.

When passing over to dimensional variables in basic formulas (9) and (12), it is necessary to take into account that, according to Eq. (5), the DW velocity $\dot{q}$ is measured in units of $4\pi\gamma M\Delta$ and that the frequency ω and the amplitude **H** of the field rotating in the basal plane are measured in units of $4\pi\gamma M$ and $8M$, respectively.

Let us compare the DW drift caused by a strong field **H** rotating in the *xy* plane and the DW motion in the usual case where a dc magnetic field H_z is applied (along the *z* axis). In the case of a dc field H_z, there is a mode of DW motion similar to that described by Eq. (9), namely, $\dot{q} = H_z/\alpha$, where the role of the frequency is played by the field H_z and the DW velocity does not exceed the Walker velocity, which is equal to 1/2 (when measured in units of $2\pi\gamma M\Delta$; $Q \gg 1$). For the mode of asynchronous precession of the DW magnetic moment that is similar to the mode described by Eq. (12), the differences are more significant. As can be seen even from Eqs. (4), in the limit of $H_z \gg 1$, the DW velocity does not tend to zero (as is the case in Eq. (13) with increasing frequency) but follows a quite different formula $\dot{q} = \alpha H_z/(1 + \alpha^2)$, which is typical for the one-dimensional case [1, 6].

There is also a fundamental difference between the effect under study and the DW drift in a weak field. In the case of DW drift in a weak field, the spins are only slightly deflected from their equilibrium direction and oscillate near it with a small amplitude. In a strong field $H > 1$, the spins in the central part of the DW are dragged by the field and rotate following the field.

In concluding this section, let us find how the results following from Eqs. (9) and (12) are affected by the magnetostatic interaction $2\pi M_y^2$, which is described by the right-hand side of Eq. (7b) and was previously omitted. The first-order correction $\delta\Phi \sim 1/H$ to the mode described by Eq. (9) with $\Phi = \Phi_0$ is given by the linear equation with constant coefficients

$$\frac{1 + \alpha^2}{\alpha}\delta\dot{\Phi} + H\cos\Phi_0\delta\Phi = -\frac{1}{2}\sin 2(\Phi_0 + \omega t), \quad (14)$$

whose steady-state solution oscillates in time as

$$\delta\Phi = C_1\cos 2\omega t + C_2\sin^2\omega t,$$
$$C_{1,2} = \frac{-(H/2)\cos\Phi_0(\sin 2\Phi_0, \cos 2\Phi_0) \pm (\alpha + \alpha^{-1})\omega(\cos 2\Phi_0, \sin 2\Phi_0)}{\det}, \quad (15)$$
$$\det = H^2\cos^2\Phi_0 + 4\omega^2(\alpha + \alpha^{-1})^2.$$

In order to determine the effect of oscillations (15) on Φ_0, we write the solution to Eq. (7b) in the form $\Phi = \Phi_0 + \delta\Phi$, where Φ_0 and $\delta\Phi$ are fast and slow functions of time, respectively. Let us expand the left-hand part of Eq. (7b) in a power series in $\delta\Phi$ and keep terms $\sim\delta\Phi^2$ (the right-hand side is small, and we can here limit ourselves to terms $\sim\Phi_0$). The fast motion obeys Eq. (14) as before. However, the equation of slow motion now contains a term $\sim\delta\Phi^2$, because the time average of the square of oscillations (15) is not zero. Instead of Eq. (9), the phase angle is given by

$$\sin\Phi_0 = -\frac{(1 + \alpha^2)\omega}{\alpha H(1 - \delta\Phi^2/2)}, \quad (16)$$

where $\overline{\delta\Phi^2} = 1/(4\det) \ll 1$ and det is given by Eq. (15). In the approximation in question, the magnitude of the limiting angle [see Eq.(9)] keeps the same value $\Phi_0 = -\pi/2$ but the limiting frequency ω_c at $H \gg 1$ gets a small correction $\sim 1/(32H(\alpha + \alpha^{-1}))$. Analysis demonstrates that, in the mode of motion described by Eqs. (10)–(12), analogous corrections are likewise small, but this is too cumbersome to present here.

4. NUMERICAL VERIFICATION

The main result of the present paper, the frequency dependence of the absolute value of the DW velocity given by Eqs. (9) and (12) (see also figure), has to be verified numerically in two respects. It is necessary to check whether (i) the right-hand side of Eqs. (7) can be neglected in a low field H and whether (ii) approximate equations (7) can be used instead of the Landau–Lifshitz equations (2b) for low values of the quality factor Q.

In the first case, the complete equation (7b) was numerically integrated (the details of this integration are not presented here). The calculations demonstrate that, for the values of H and α given in the figure, the difference between the exact and approximate solutions is small. Moreover, since the amplitude of oscillations $\delta\Phi$ in Eq. (15) is small, the typical peak in the $V(\omega)$ dependence is preserved up to $H \sim 1$. Near this peak, we have $\Phi_0 \approx -\pi/2$, $\omega \approx \omega_c \approx \alpha H$ ($\alpha < 1$), and $\delta\Phi \approx \alpha/(4\omega_c) \approx 1/(4H)$ [see Eqs. (9), (14)]. The numerical calculations show that this peak is still quite distinct even at $H = 2$ (here, $\delta\Phi \sim 0.1$) and that its shift in frequency is very small, in agreement with Eq. (16).

The second case requires numeric integration of the Landau–Lifshitz equations (2). Numeric calculations were performed for several values of the quality factor $Q > 1$, which does not enter into the Slonczewski equations (7). The results of integrating Eqs. (2) for three typical values of Q make it possible to draw certain conclusions about the accuracy of the approximate theory based on Eqs. (7) and are shown in the figure.

Overall, despite some scatter, the values calculated from Eqs. (2) are quite close to Eqs. (9) and (12) (shown by solid lines in the figure) derived in the approximate Slonczewski theory. The discordance (in agreement with the general theory [14, 15]) increases with decreasing Q. The results of calculations for $H =$

10 and $Q = 10$ are not shown in the figure, because they can hardly be compared to the approximate theory. Indeed, Eqs. (7) are valid for $H/H_a \ll 1$. In dimensionless units (5), this condition has the form $2H/(\pi Q) \ll 1$. For $H = 10$ and $Q = 10$, the left-hand side of this inequality is $2/\pi \approx 0.7$, which is certainly not sufficiently small as compared to unity.

5. DISCUSSION

It has been demonstrated in this paper that a sufficiently strong magnetic field rotating in the basal plane of a uniaxial ferromagnet drags (with some lagging in phase) the magnetic moments located near the center of the domain wall. Qualitatively, the mechanism of translational DW motion can be described using the fundamental equations (2b) rewritten in angular variables (the polar angle θ of the magnetization vector **M** is measured from the z axis and the azimuth angle ψ, from the x axis in the xy plane):

$$\begin{aligned} \dot{\psi}\sin\theta - \alpha\dot{\theta} &= \frac{\gamma}{M}\delta w/\delta\theta, \\ \dot{\theta}\sin\theta + \alpha\dot{\psi}\sin^2\theta &= \frac{-\gamma}{M}\delta w/\delta\psi. \end{aligned} \tag{17}$$

It is clear that, near $\theta \approx 0$ (which corresponds to the DW center) it follows from the first of Eqs. (17) that $\dot{\psi}\sin\theta - \alpha\dot{\theta} = 0$, because $\delta w/\delta\theta \approx 0$. (In the case where a field H_z is applied, the zero in the right-hand side of the above equations should be replaced by γH_z.) In the regime of translational motion of the DW described by Eq. (3), which is due to the spin-drag effect described by the second of Eqs. (17), we have $\dot{\psi} \approx \omega$. Therefore, $\dot{q} = -\mathrm{sgn}(\theta')\omega\Delta/\alpha$, which coincides with Eq. (9) obtained previously from the Slonczewski equations. So, in the case under consideration, the translational DW motion is due to the balance of the kinetic moment and the dissipative action of the magnetic moments in the center of the DW in the rotating field **H**.

For the sake of completeness and comparison with the results of the present work, we give here the DW drift velocities in a weak circularly polarized field **H** calculated within the most consistent, in our opinion, approach [5]. The results are expressed in dimensionless variables (5) and expanded where necessary in power series in $1/Q$. Below the frequency of uniform FMR ($\omega < \gamma H_a$), for a weak field ($H < 8M$), it follows from [5, Eq. (40)] that, in dimensionless units, $V = \omega H^2/(2\alpha Q)$ for $\omega < Q$. For $\omega > \gamma H_a$, it follows from [5, Eq. (39)] that $V = 2H^2/(\alpha\omega)$ for $\omega > Q$. In both cases, the DW drift is due to the magnetic pressure exerted on the DW that appears because of the Larmor precession of the spins inside domains [1, 2] (this precession is not described by the Slonczewski equations [14, 15]). This effect is small ($\sim 1/Q$) everywhere except in the region near the uniform FMR $\omega \sim \gamma H_a$ [2].

Let us dwell now on the applicability of the results obtained here to DWs in films with $Q > 1$ that are magnetized normal to their surface. Among rare-earth ferrite-garnets films, which are popular objects of experimental studies, the Bi-based compounds seem to be the most suitable. For them, the values of Q can be higher than 50, which makes it possible to realize the strong-field regime characterized by inequalities (6).

A particular feature of normally magnetized films is that the DWs in them are not one-dimensional but rather are distorted along the z axis (i.e., through the film thickness h) by the magnetostatic field $H_y(y = q, z) = \mathrm{arctanh}(2z/h)$ directed perpendicular to the DW surface. This field is created by magnetic poles located on both surfaces of the film. The resulting DW is called twisted, and its properties can be described by the following version of the Slonczewski equations, which is more complete than Eqs. (4):

$$\dot{\psi} - H_z + \alpha\dot{q} = \varepsilon^2 q'', \tag{18a}$$

$$\begin{aligned} \dot{q} - \alpha\dot{\psi} = -\varepsilon^2\psi'' + [\sin\psi - H_y(z)]\cos\psi \\ + H_x\sin\psi - H_y\cos\psi \end{aligned} \tag{18b}$$

with the boundary conditions on both surfaces of the film $q'(z = \pm 1) = \psi'(z = \pm 1) = 0$. In these equations, we use the same notation as in Eqs. (4) but take into account the dependence on z (z is measured in units of $h/2$; the derivatives with respect to the dimensionless coordinate z are marked by primes; $\varepsilon = 2\Lambda/h$; $\Lambda = \Delta Q^{1/2}$ and Δ are the width parameters of the Bloch line and DW, respectively).

The applicability of the results obtained in this work to DWs in normally magnetized films depends on how much the DWs are twisted [i.e., how large their deviation from the simple Bloch structure described by Eq. (3) is]. In this connection, it should be noted that, as follows from Eqs. (18), the nonuniform exchange interaction, which is proportional to $\varepsilon^2\psi''$, substantially suppresses the effect of the field $H_y(z)$ and impedes nonuniform rotation of the spins along the z axis if the film thickness is small, i.e. for $\varepsilon > 1$. This fact was first found quite long ago using numerical integration of Eqs. (18) (A. Hubert, 1975; see a detailed rendition of his work in [1]), and numerical results have shown that the twist is suppressed even at values of ε as small as ≈0.8.

An appropriate perturbation theory, in which the small parameter is $1/\varepsilon^2$ and the boundary conditions are of importance, was developed in [16, 17]. If we integrate Eqs. (18) over the film thickness (≥ z ≥), then, taking into account the boundary conditions and antisymmetry of the field $H_y(z)$, we immediately get the one-dimensional equations (4). The next terms in the expansion in a power series in $1/\varepsilon^2$ (both dependent and independent of z) can be obtained using the perturbation theory developed in [16, 17]. In this theory, the one-dimensional dynamic equations play the role of the solvability conditions for two-dimensional problems that take into account a weak twist.

Concerning the case of thick films, we note that the strong-field criterion (6), which is a necessary (but not sufficient) condition for realization of the DW drift mechanism considered here, is significantly altered. The suppression of the DW twist and transition to the simple Bloch (or Néel) structure in thick films (i.e., for $\varepsilon < 1$) depend on the field direction and, for certain field orientations, occur when the field magnitude significantly exceeds $8M$, as follows from theoretical considerations [18, 19] and experiment [20].

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SPELL: 1. astroid