

**SHORT  
COMMUNICATIONS**

## Bloch Lines in Domain Walls of Antiferromagnets

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**Abstract**—Approximants describing the substructure of 180° domain walls in antiferromagnets with a high uniaxial anisotropy are derived. The structure and spectrum of Bloch lines separating the parts of domain walls with oppositely directed antiferromagnetic vectors are determined.

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In magnetics, Bloch lines (BLs) divide the domain wall (DW) surface into subdomains and significantly influence the DW properties. Numerous attempts have been made to construct ferromagnetic memory devices based on Bloch lines [1, 2].

BLs in ferromagnets with a high uniaxial anisotropy seem to have been studied most fully to date [3]. In weakly anisotropic films, the respective study was pioneered much earlier [4] (for the state of the art in this field, see [5]). In cubic ferromagnets, BLs are observed irrespective of whether the anisotropy constant is negative [6, Fig. 1] or positive [7]. Theoretical models of BLs in weak ferromagnets are suggested in [8, 9]. Chetkin et al. [10] observed local sags in BLs moving with a high velocity in yttrium orthoferrite and related them to the vorticity displacement along the DW.

While the presence of DWs in antiferromagnets has known for long, the DW structure in antiferromagnets appears to be poorly understood. Disclinations and vorticity in antiferromagnets are analyzed in [11]. In [12], a number of multidimensional vortical solutions to the Andreev–Marchenko equations [13] for a uniaxial antiferromagnet were found. Unfortunately, among the solutions found in [12], the simplest element of the DW substructure, the localized BL similar to 180° BLs observed in uniaxial ferromagnets [3], is lacking. Below, the author, based on the Andreev–Marchenko equations [13] (see also [11]), derives reduced equations similar the Slonczewski equations [3] for ferromagnets.

Suppose that the anisotropy axis in a ferromagnet is aligned with the  $Oz$  axis and 180° DWs occupy the  $xOz$  axis. According to [13], the Lagrangian of the system,  $L = L_0 + L_1$ , is expressed through unit antiferromagnetic vector  $\mathbf{I}(r, t)$  as

$$L_0/(4M_0^2\beta_1) = -\frac{1}{2}[(d\mathbf{I}/dy)^2 - I_z^2], \quad (1.1)$$

$$L_1/(4M_0^2\beta_1) = \dot{\mathbf{I}}^2/2 - \mathbf{I} \cdot [\mathbf{I} \times \mathbf{H}] - \frac{1}{2}[(\mathbf{I} \times \mathbf{H})^2 + (\nabla_{x,z}\mathbf{I})^2 + \frac{\beta_0}{\beta_1}I_y^2]. \quad (1.2)$$

In (1), kinetics contributions (proportional to  $\sim \dot{\mathbf{I}}$ ), nonuniform exchange (constant  $\alpha$ ), uniaxial and rhombic anisotropies (constants  $\sim \beta_1$  and  $\sim \beta_0$ ), and external magnetic field  $\mathbf{H}$  are taken into account. The lengths are measured in units of the DW width,  $\Delta = (\alpha/\beta_1)^{1/2}$ ; the time, in units of  $1/\omega_0$ , where  $\omega_0 = 1/2\gamma M_0(\alpha\beta_1)^{1/2}$  ( $M_0$  is the lattice rated magnetization,  $\gamma$  is the gyromagnetic ratio, and  $a \sim 10^3$  is the dimensional constant of antiferromagnetic interaction); and the magnetic fields, in units of  $H_0 = \omega_0/\gamma$ .

The only strained point in this work is that Lagrangian  $L_0$ , which specifies 180° DWs of the antiferromagnet, is used as the initial one, since it is invariant in the  $xOy$  basal plane of the antiferromagnet. Parametrizing the antiferromagnetic vector in the form  $\mathbf{I} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ , we obtain from  $L_0$

$$\cos\theta_0 = -\tanh[(y - q(x, z, t))/\Delta], \quad \varphi_0 = \psi(x, z, t). \quad (2)$$

The desired equations for DW center ( $q(x, z, t)$ ) and azimuthal angle  $\psi(x, z, t)$  on the DW surface are determined by perturbation  $L_1$ . As follows from (1.2), such a statement of the problem is correct if  $\omega \ll \omega_0$  and  $H \ll H_0$ . In the DW plane, the characteristic lengths exceed  $\Delta$  and  $\beta_1 \ll \beta_0$ .

In such a statement, our problem is similar to the problem of derivation of the Slonczewski equations for  $q(x, z, t)$  and  $\psi(x, z, t)$  in highly anisotropic ferromagnets (here, the condition  $\beta_1/\beta_0 \gg 1$  replaces the condition  $Q \gg 1$ , where  $Q$  is the figure of merit). These equations were asymptotically derived from the Landau–Lifshitz equations by expanding in powers of  $1/Q$  in

[14, 15]. It is convenient to apply the Lagrangian statement as in [15], where it was shown that the desired equations can also be obtained by substituting the starting equations into the Lagrangian ((2) into (1.2) in our case), integrating the result over  $y$  in infinite limits, and only then varying in  $q$  and  $\psi$ . Taking into account the contribution from dissipative function  $R = \alpha_G M_0 \mathbf{I}^2 / 2\gamma$  (where  $\alpha_G$  is the Gilbert damping constant) in a similar way, we arrive at the effective equations

$$\begin{aligned} & \ddot{q} + \sqrt{a/\beta_1} \alpha_G \dot{q} - \nabla_{x,z}^2 q \\ &= \frac{\pi}{2} \frac{d}{dt} (H_x \sin \psi - H_y \cos \psi), \end{aligned} \quad (3.1)$$

$$\begin{aligned} & \ddot{\psi} + \sqrt{a/\beta_1} \alpha_G \dot{\psi} - \nabla_{x,z}^2 \psi + (\beta_0/\beta_1) \sin \psi \cos \psi \\ &+ (H_x \cos \psi + H_y \sin \psi) (-H_x \sin \psi + H_y \cos \psi) \\ &= -\frac{\pi}{2} \dot{q} (H_x \cos \psi + H_y \sin \psi) + \dot{H}_z. \end{aligned}$$

Let a BL separate two subdomains with  $\psi(x \rightarrow \mp\infty) = 0, \pi$  on the  $Ox$  axis. This is possible if  $H_x^2 < H_s^2 \equiv \beta_0/\beta_1$  (at  $H_x^2 > H_s^2$ ,  $\psi(\pm\infty) = \pm\pi/2$ ), where  $H_s$  is the switching field of the structure, which is  $(\beta_0/\beta_1)^{1/2}$  times lower than  $H_0 = 2M_0(a\beta_1)^{1/2}$ . Then, considering the case  $H_y = 0$ , we find from (3.2) that

$$\cos \psi_0 = -\tanh[(x - X)/\Lambda]. \quad \Lambda = \Delta/(H_s^2 - H_x^2)^{1/2}. \quad (4)$$

If the BL moves freely with constant velocity  $V$ , solution (4) retains the form but now we have

$$\frac{1}{\Lambda} \rightarrow \frac{1}{\Lambda_d} \equiv \left( \beta_0/\beta_1 - H_x^2 - \left( \frac{\pi}{2} \right)^2 \frac{V^2 H_x^2}{1 - V^2} \right)^{1/2} (1 - V^2)^{-1/2}. \quad (5)$$

As follows from (5), at  $H_x^2 \ll \beta_0/\beta_1 < 1$ , the limit value of velocity  $V_0$  is close to unity (in dimensional units,  $V_0 \equiv \omega_0 \Delta$ ). The motion of the BL is accompanied by the occurrence of a step on the DW. The height of this step depends on the velocity,

$$\frac{q - q_0}{\Delta} = \frac{\pi V_0 V}{V_0^2 - V^2} \frac{H_x}{H_0} \Lambda_d \arctan \exp\left(\frac{x - Vt}{\Delta \Lambda_d}\right), \quad (6)$$

where  $q_0$  is a constant. Note also that, in the limit  $H_{x,y} = 0$ , system (3) splits into two independent equations: a linear equation for  $q(x, t)$  and a sin-Gordon equation with external pumping  $\sim \dot{H}_z(t)$  for  $\psi(x, t)$ .

To conclude, consider linear oscillations of the BL in the presence of weak permanent field  $H_y$ . For small amplitudes  $\delta q$  and  $\delta \psi$ , we have from (3)

$$(-\omega^2 + \omega_q^2) \delta q - \delta q'' = -i \frac{\pi}{2} \omega H_y \sin \psi_0 \delta \psi, \quad (7.1)$$

$$(-\omega^2 + \omega_\psi^2) \delta \psi + \hat{L} \delta \psi = i \frac{\pi}{2} \omega H_y \sin \psi_0 \delta q - i \omega H_z, \quad (7.2)$$

$$\hat{L} = -\frac{d^2}{dx^2} + (\beta_0/\beta_1) \cos 2\psi_0. \quad (7.3)$$

Along with eigenfrequency  $\omega$ , system (7) includes quasi-elastic frequencies  $\omega_q$  and  $\omega_\psi$  of the DW and BL. As follows from (7), the DW oscillation spectrum contains two branches of bending oscillations,  $\omega^2 = \omega_q^2 + k^2$  and  $\omega^2 = \omega_\psi^2 + \beta_0/\beta_1 + k^2$ , localized outside the BL.

The theory of BL free oscillations in ferromagnets is based on the fact that, at  $\omega < \omega_q < 1$ , highly delocalized sags arise on a DW near a BL. According to (7.1), the same is true for DWs in antiferromagnets when the DW sag,  $\lambda = (\omega_q^2 - \omega^2)^{-1/2}$ , exceeds width  $\lambda$  of the BL,

$$\delta q(x) = i \frac{\pi}{2} \omega H_y \lambda \exp[-|x|/\lambda] \delta X. \quad (8)$$

It should also be noted that the lower eigenfunction of operator  $\hat{L}$ ,  $\delta \chi = \sin \psi_0$ , is, in this case, a shear mode of the BL along the DW; therefore,  $\hat{L} \delta \chi = 0$ . Substituting (8) into (7.2) and taking advantage of the fact that, according to (4),  $\delta \psi = -\delta X \sin \psi_0 / \Lambda$  for the shear mode, we leave only terms containing  $\delta \chi$  in expansion (7.2) in eigenfunctions. Eventually, we arrive at the dispersion relation

$$(\omega_\psi^2 - \omega^2)(\omega_q^2 - \omega^2)^{1/2} = (\pi \omega H_y / 2)^2 (\beta_1 / \beta_0)^{1/2}. \quad (9)$$

The only solution to (9) lies in the domain  $\omega \leq \min(\omega_q, \omega_\psi)$  and differs little from  $\omega_q$  or  $\omega_\psi$  at  $H_y^2 \ll 1$ . Assuming that the quasi-elastic frequencies are completely defined by magnetoelastic energy  $w_{me} \sim b^2/c$  (where  $a$  and  $b$  are the estimated shear modulus and magnetoelastic constant), we find that  $\omega_q \sim \omega_\psi \sim w_0(w_{me}/\beta_1 M_0^2)^{1/2}$ . For typical orders of the parameters involved ( $M_0 \sim 10^2$  G,  $\beta_1 \sim 10$ ,  $a \sim 10^3$ ,  $\gamma \sim 10^7$  (s Oe) $^{-1}$ ,  $c \sim 10^{12}$ , and  $b \sim 10^7$  erg/cm $^3$ ), we have  $\omega_0 \sim 10^{11}$  s $^{-1}$  and  $\omega_q \sim \omega_\psi \leq \omega_0/10$ .

If  $H_y = 0$ , field  $H_z(t)$  entering into (7.2) does not cause sags on the DW but sets the BL in motion. The BL velocity can be found by expanding (7.2) in eigenfunction  $\delta \chi$  as above,  $\dot{X} = -\pi \Lambda \dot{H}_z / (4\alpha_G a M_0)$ . For the above orders of the parameters,  $\Delta \sim 10^{-5}$  cm, and  $\beta_0 \sim 1$ , it is necessary that  $\dot{H}_z$  be on the order of  $10^6$  Oe/s to have  $\dot{X} = 1$  cm/s even if  $\alpha_G \sim 10^{-4}$ .

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