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THEORY OF METALS

Two-Frequency Excitation of a Domain-Wall Drift in Uniaxial Ferromagnets with a Large Anisotropy Constant

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Abstract—Using Slonczewski equations, drift motion of a domain wall (DW) has been considered in strong magnetic fields, when one of the field components is polarized along the easy axis (EA) and the second, in the basal plane of the ferromagnet. A two-frequency mechanism of excitation is suggested which differs from the usual single-frequency mechanism in that the frequencies of the exciting fields differ in magnitude (although are linked by a certain relationship). For the above configuration of fields, the single-frequency mechanism leads to a drift velocity which vanishes in the limit of large quality factors, whereas the two-frequency mechanism yields a finite contribution in strong fields.

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1. INTRODUCTION

By the drift of a domain wall (DW) in ferromagnetic samples, the phenomenon of its average translational motion under the effect of oscillating magnetic fields is usually meant. The drift of DWs and the closely related effect of reorientation of the system of DWs is treated in numerous works (predominantly, concerning uniaxial ferromagnets); a review of data published up to 1979 can be found in [1]. To excite drift, two orthogonal magnetic fields oscillating with the same frequency are usually applied to the ferromagnet (single-frequency excitation). The theoretical description of this effect (see [2–4] for the case of uniaxial ferromagnets) is, as a rule, restricted to a quadratic approximation in small amplitudes of the exciting fields.

The transition into practically interesting range of strong fields was realized in [5], which proved to be possible at the expense of a narrowing of the class of magnets to be considered. For uniaxial ferromagnets, which are characterized by a large quality factor

$$Q \equiv H_a/4\pi M \gg 1, \quad (1)$$

where H_a is the effective anisotropy field and M is the saturation magnetization, instead of the Landau–Lifshitz equations, much simpler Slonczewski equations [1] can be used for the 180° DW (and only for this DW). In this case, the effects that are proportional to ~$1/Q$ are omitted; their consideration should be restricted to the range of weak magnetic fields $H \ll H_a$ and low frequencies $\omega \ll \omega_0$, where ω_0 is the frequency of natural ferromagnetic resonance (see, e.g., [6]). In our previous work [5], we found, in terms of the Slonczewski equations, a nonlinear solution to the problem of single-frequency drift of a DW, where the strong exciting field is oriented and rotates in the basal plane of the magnet.

It seemed natural to apply the approach used in [5] to another frequently used configuration, where one of the external fields is, as before, polarized in the basal plane and the second field oscillates along the EA of the ferromagnet. However, as follows from Slonczewski equations, in the single-frequency approach the effect of drift in such a configuration is absent (see Sections 2 and 3). For true, a more rigorous consideration using Landau–Lifshitz equations (see, e.g., [4]) shows that in this case even in the quadratic approach there exists some drift which is not "felt" by the Slonczewski equations since it is proportional to ~$1/Q$.

Nevertheless, in the above configuration, as is shown in this work, in strong magnetic fields there does exist a significant drift of a DW which does not vanish even in the limit of $Q \longrightarrow \infty$. To reveal it, it is sufficient to change the conditions of excitation and pass from the single-frequency to two-frequency excitation, where two orthogonal polarizations of the magnetic field oscillate with different frequencies linked by a certain relationship.

2. PERTURBATION THEORY AND ANALYTICAL RESULTS

Let us consider a uniaxial ferromagnet satisfying condition (1) whose EA is collinear to the $0z$ axis and whose basal plane coincides with the $x0y$ plane. Let the plane of the 180° DW is the xOz plane, the magnetization distribution $\mathbf{M}(y - q(t), t)$ (where $q(t)$ is the position of the DW center) depends on the coordinate y and time t, and the boundary conditions are written as $\mathbf{M}(y \longrightarrow \pm\infty, t) \longrightarrow \mp M\mathbf{e}_z$. The Slonczewski equations are a reduced form of the Landau–Lifshitz equations; they relate the position of the DW center $q(t)$ and the azimuthal angle $\psi(t)$ (which is measured from the 0x axis) of the vector $\mathbf{M}(y - q(t)t)$ at the center of the DW $y = q(t)$. With allowance for the Zeeman term $-\mathbf{HM}$ ($\mathbf{H}$ is the magnetic-field vector) and magnetostatic term $2\pi M_y^2$, they take on a simple form

$$\dot{\psi} + a\dot{q} = H_z(t), \tag{2.1}$$

$$\dot{q} - \alpha\dot{\psi} = (\sin\psi - H_y(t))\cos\psi + H_x(t)\sin\psi. \tag{2.2}$$

Equations (2) contain dimensionless variables (to the right of the arrows)

$$t \longrightarrow t/(4\pi\gamma M), \quad q \longrightarrow \Delta q,$$
$$H_z \longrightarrow 4\pi M H_z, \quad H_{x,y} \longrightarrow 8MH_{x,y}, \tag{3}$$

where $\gamma > 0$ is the magnetomechanical ratio and $\alpha > 0$ is the Gilbert damping factor. The fields that excite the DW drift may be written as

$$H_z(t) = H_{0z}\cos(\omega_1 t + \omega_{0z}),$$
$$H_y(t) = H_{0y}\cos(\omega_2 t + \omega_{0y}), \tag{4}$$

where $\omega_{1,2}$, $H_{0z,0y}$, and $\omega_{0z,0y}$ are the frequencies, amplitudes, and phases of the fields. The $H_x(t)$ field is not considered below, since its treatment is analogous to that of $H_y(t)$ and leads to similar results. Equations (2) reduce to a single equation of the first order

$$\dot{\psi}(\tau) = \frac{1}{\omega_2(1+\alpha^2)}[H_{0z}\cos(\omega_1\tau/\omega_2 + \Phi)$$
$$+ \alpha(H_{0y}\sin\tau\cos\psi - \sin\psi\cos\psi)], \tag{5.1}$$

in which

$$\tau = \omega_2 t + \varphi_{0y}, \quad \Phi = \varphi_{0z} - \omega_1\varphi_{0y}/\omega_2. \tag{5.2}$$

By determining $\psi(\tau)$ from Eq. (5.1), we find $\dot{q}$ from the linear equation (2.1).

The main problem is to calculate the parameter Ω of the solution of the type $\psi(\tau) = \Omega\tau + \eta(\tau)$ of the nonautonomous nonlinear equation (5). Proceeding from the representation of the magnetization in the form $\mathbf{M}(y = q(t), t) = M(\cos\psi(\tau)\mathbf{e}_x \sin\psi(\tau)\mathbf{e}_y, 0)$ at the center of the DW, we can interpret Ω as the frequency of the rotating motion of $\mathbf{M}(y = q(t), t)$, and $\eta(\tau)$, as oscillations (libration), which are superimposed onto this motion, with a zero average in time $\langle\eta(\tau)\rangle_\tau$. The drift velocity is determined by averaging over the time in Eq. (2.1) ($\langle\dot{\psi}(t)\rangle_t \equiv \omega_2\langle\dot{\psi}(\tau)\rangle_\tau$; for the relation between t and τ, see Eq. (5.2)):

$$\langle\dot{q}(t)\rangle_t = -\langle\dot{\psi}(t)\rangle_t/\alpha = -\Omega/\alpha. \tag{6}$$

Note at once that the sought-for drift solution exists only in the presence of both fields, $H_z(t)$ and $H_y(t)$, in (5.1) and of a certain link between ω_1 and ω_2. (In the case of the same polarization fields, the most interesting phenomena at the unequal excitation frequencies appear to occur upon the superimposition of the dc and ac components of the field H_z (see [3, 7]).

When constructing a perturbation theory for Eq. (5), we assume that the contribution from the field H_z prevails over the contributions from H_y and from magnetostatic energy. In this case, in the zero approximation we obtain a purely vibrational (librational) solution (5.1)

$$\psi^{(0)}(\tau) = b\sin(\omega_1\tau/\omega_2 + \Phi),$$
$$b = H_{0z}/\omega_1(1+\alpha^2). \tag{7}$$

Representing the sought-for solution in the form $\psi(\tau) = \Omega\tau + \nu(\tau)$, we transform (2.1) into an equation in which the right-hand side is proportional to some small parameter and admits averaging over the "fast" motion:

$$\dot{\nu}(\tau) = \frac{\alpha}{\omega_2(1+\alpha^2)}\Big[(H_{0y}\sin\tau\cos[\psi^{(0)}(\tau) + \nu(\tau)]$$
$$-\frac{1}{2}\sin 2[\psi^{(0)}(\tau) + \nu(\tau)]\Big]. \tag{8}$$

The solution to (8) is sought for in the form of the so-called straightforward expansion [8], which reduces to

$$\nu(\tau) = \nu^{(1)}(\tau) + \nu^{(2)}(\tau) + \ldots, \tag{9}$$

and does not require the introduction of auxiliary parameters into the equations and into their expansions. However, Eq. (9) permits us to determine only the first vanishing approximation for Ω, which arises upon the appearance of a constant, nonoscillating term in the expansion of the right-hand side of Eq. (8) and which yields the sought-for contribution proportional to $\sim\Omega t$. Naturally, in the following order the expansion (9) becomes invalid, since it contains terms which diverge at $t \longrightarrow \infty$ stronger than $\sim t$. For our further considerations, only two first approximations are sufficient:

$$\dot{\nu}^{(1)}(\tau) = \frac{\alpha}{\omega_2(1+\alpha^2)}$$
$$\times\Big[(H_{0y}\sin\tau\cos\psi^{(0)}(\tau) - \frac{1}{2}\sin 2\psi^{(0)}(\tau)\Big]; \tag{10.1}$$

$$\dot{\nu}^{(2)}(\tau) = \frac{-\alpha}{\omega_2(1+\alpha^2)}$$
$$\times\Big[(H_{0y}\sin\tau\sin\psi^{(0)}(\tau) + \frac{1}{2}\cos 2\psi^{(0)}(\tau)\Big]\nu^{(1)}(\tau), \tag{10.2}$$

in which $\psi^{(0)}(\tau)$ is determined by Eq. (7). This system can easily be integrated, since its right-hand sides are successively determined by preceding equations. By using the well-known expansions into Fourier series

$$\cos(z\sin\vartheta) = J_0(z) + 2\sum_{k=1}^{\infty} J_{2k}(z)\cos 2k\vartheta, \tag{11.1}$$

$$\sin(z\sin\vartheta) = 2\sum_{k=1}^{\infty} J_{2k+1}(z)\cos(2k+1))\vartheta, \quad (11.2)$$

which contain Bessel functions, we first find a solution to (10.1) as

$$\begin{aligned}\nu^{(1)}(\tau) = {} & \frac{\alpha}{\omega_2(1+\alpha^2)}\Big\{-H_{0y}\Big[J_0(b)\cos\tau \\ & + \sum_{k=1}^{\infty} J_{2k}(b)\Big(\frac{\cos[(1-2k\omega_1/\omega_2)\tau - 2k\Phi]}{1-2k\omega_1/\omega_2} \\ & + \frac{\cos[(1+2k\omega_1/\omega_2)\tau + 2k\Phi]}{1+2k\omega_1/\omega_2}\Big)\Big] \\ & + \frac{1}{2}\sum_{k=1}^{\infty} J_{2k+1}(2b)\frac{\cos[(1+2k)(\omega_1\tau/\omega_2+\Phi)]}{(2k+1)\omega_1/\omega_2}\Big\},\end{aligned} \quad (12)$$

where b is determined by Eq. (7), and Φ, by Eq. (5.2).

One feature (and restriction) of the straightforward expansions is the appearance in them (in this or that approximation) of terms which diverge at $t \longrightarrow \infty$ (which have already been mentioned above) and also of resonance denominators. In the case of Eq. (12), the divergences arise when $\omega_2 = 2k\omega_1$, $k = 1, 2, \ldots$, but, except for the frequencies indicated, the purely oscillating solution (12) remains valid. However, already the equation of the next, second order (Eq. (10.2)) after substitution of (12) into its right-hand side leads to a nonzero velocity of rotation Ω. It can easily be found that this occurs if we impose the following restriction on the frequencies of the fields $H_z(\omega_1 t)$ and $H_y(\omega_2 t)$:

$$\omega_1 = 2\omega_2; \quad (13)$$

it is precisely this restriction that serve as the condition for the two-frequency excitation of the DW drift. Note also that no single-frequency drift ($\omega_1 = \omega_2$) arises in this case. Using the Fourier series (11) and trigonometric formulas (represent of the product of harmonic functions through the sum of individual harmonic terms), we find that the only contribution to the right-hand side which forms $\Omega \neq 0$ can come only from terms that are quadratic in H_{0y}. The contributions that are linear in H_{0y}, as well as the contribution of the magnetostatic energy (proportional to $\sim J_{2k+1}/(b)$ in (12)) in the second-order perturbation theory, are purely harmonic, so that the effect of drift is proportional to $\sim H_{0y}^2$.

Now, we write down the right-hand side of Eq. (10.2) that is responsible for the condition $\Omega \neq 0$:

$$\begin{aligned}&\Big(\frac{\alpha H_{0y}}{\omega_2(1+\alpha^2)}\Big)^2\{J_0(b)J_1(b)\sin 2\tau\sin(2\tau+\Phi) \\ & + \sin\tau\sum_{m=0,k=1}^{\infty} J_{2k}(b)J_{2m}(b) \\ & \times\Big[\frac{\sin[(2m+1)(2\tau+\Phi)-(1-4k)\tau+2k\Phi]}{1-4k} \\ & + \frac{\sin[(2m+1)(2\tau+\Phi)+(1-4k)\tau-2k\Phi]}{1-4k} \\ & + \frac{\sin[(2m+1)(2\tau+\Phi)-(1+4k)\tau-2k\Phi]}{1+4k} \\ & + \frac{\sin[(2m+1)(2\tau+\Phi)+(1+4k)\tau+2k\Phi]}{1+4k}\Big]\Big\}.\end{aligned} \quad (14)$$

After averaging over time τ, the first term in braces in Eq. (14) is proportional to $\sim J_0(b)J_1(b)\cos\Phi$. Under the sign of sum with respect to m and k in (14), we should separate terms that do not vanish upon averaging in time. The criterion for this choice is the proportionality of some terms in the brackets to a common factor $\sin\tau$ standing before the sign of sum. The necessary contributions come only from the second term at $k = m + 1$ and the last term at $k = m$ ($k = 1, 2, \ldots$). The double sum now can be convoluted with respect to one of the indices to obtain, instead of (14), the following expression:

$$\begin{aligned}&\Big(\frac{\alpha H_{0y}}{\omega_2(1+\alpha^2)}\Big)^2\cos\Phi\Big\{J_0(b)J_1(b)\sin^2 2\tau + \sin^2\tau \\ & \times\Big[\sum_{k=0}^{\infty}\frac{J_{2k+1}(b)J_{2k+2}(b)}{3+4k} + \sum_{k=1}^{\infty}\frac{J_{2k}(b)J_{2k+1}(b)}{1+4k}\Big]\Big\}.\end{aligned} \quad (15)$$

After averaging over the initial time t (for the relation between the averaging over t and τ, see (6)) and regrouping of terms under the sign of sum, we obtain the sought-for frequency of rotation as

$$\begin{aligned}\Omega|_{\omega_1=2\omega_2} \simeq \langle\dot{\nu}^{(2)}\rangle_t = {} & \frac{\alpha^2 H_{0y}^2\cos\Phi}{2(1+\alpha^2)^2\omega_2} \\ \times\sum_{m=0}^{\infty}\frac{J_m(b)J_{m+1}(b)}{2m+1} = {} & \frac{\alpha^2 H_{0y}^2\cos\Phi}{2(1+\alpha^2)^2\omega_2}\frac{\sin^2 2b}{2b}.\end{aligned} \quad (16)$$

Summation of the infinite series of products of Bessel functions, which leads to a nonnegative sum, was realized using formula 5.7.13.2 from the handbook [9]. We remind that upon the passage to dimensional units, the frequency, velocity, and field H_{0y} are measured in units of $4\pi\gamma M$, $4\pi\gamma M\Delta$, and $8M$; $b = \gamma H_{0z}/[\omega_1(1+\alpha^2)]$, and $\Phi = \varphi_{0z} - 2\varphi_{0y}$.

The velocity of the DW drift is expressed through Ω via formula (6), which in dimensional units takes on the form $\langle\dot{q}(t)\rangle_t = -\Delta\Omega/\alpha$, where Δ is the parameter of the

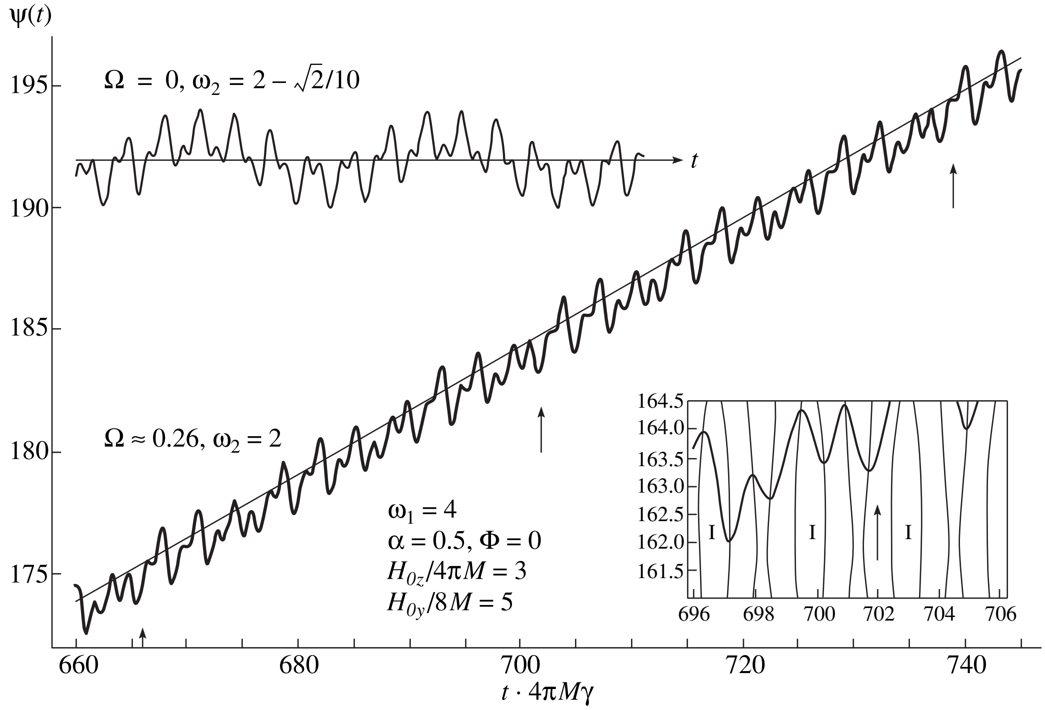

Fig. 1. Two types of solutions to Eqs. (2) (see the main text).

width of the 180° DW. A comparison of this formula with the DW velocity $\dot{q} = \Delta\gamma H_z/\alpha$ in a dc field $H_z < H_W = 2\pi M\alpha$ (H_W is the Walker velocity), which is collinear with the EA, establishes the equivalence of Ω and γH_z. For a DW could be shifted, the external field H_z should exceed the coercive force H_c (in iron garnet films, this is a magnitude of ≥0.1 Oe), which implies the fulfillment of the condition $\Omega > \gamma H_c$, where γ is the magnetomechanical ratio. This condition also refers, to the full measure, to the results of [5].

3. NUMERICAL RESULTS AND DISCUSSION

In the dynamics of DWs, which is described by a nonautonomous nonlinear equation of relaxation type (5), an important role belongs to the sign of its right-hand side. If we denote it by $f(\tau, \psi)$, then the regions of a monotonic increase (decrease) of ψ with increasing τ are determined by the conditions $f > 0$ ($f < 0$), respectively. The $f(\tau, \psi)$ function is periodic in ψ and if the ratio ω_1/ω_2 is rational, it is also periodic in τ. At the contour $f(\tau_0, \psi_0) = 0$ the oscillating solution (5) reaches an extremum; the solution $\delta\psi(\delta\tau)$ in a small vicinity of any point of the contour, as follows from the equation

$$\frac{d(\delta\psi)}{d(\delta\tau)} = f[\psi(\tau_0) + \dot{\psi}(\tau_0)\delta\tau, \tau_0 + \delta\tau] \approx \frac{\partial f}{\partial \tau_0}\delta\tau, \quad (17)$$

has the form $\delta\psi \sim \pm\delta\tau^2$. The main problem is to find under what conditions imposed on $f(\tau, \psi)$ the oscillating solution (5.1) contains, along with an oscillating part, a linearly increasing term proportional to $\sim\Omega t$.

In this work, we obtained an analytical formula for Ω (see Eq. (16)) which refers to the case where the frequencies of the orthogonal fields are linked by the condition of the two-frequency excitation of the DW drift (Eq. (13)) and one of them, $H_z(t)$, is large and is assumed to be dominating. The limitation of this approach (the straightforward expansion of (Eq. (9)), which does not permit us, while calculating Ω, to go farther than the second-order approximation, requires the verification of Eq. (16) by numerically solving the main equations (2).

The central part of Fig. 1 displays the solution $\psi(t)$ to Eq. (2) (solid curve) in an arbitrarily chosen time interval for the values of the parameters indicated nearby. The figure also contains a straight line Ωt whose slope $\Omega = 0.26\ldots$ was determined from the total slope of the curve $\psi(t)$ and agrees with the theoretical value calculated from (16) to an accuracy of two decimal places. The fine structure of a fragment of the $\psi(t)$ curve is shown in the lower inset in the right-hand bottom corner against the background of periodic contours of $f(\tau_0, \psi_0) = 0$ (small vertical ellipses at which $f = -4.3$ outline the regions where $\psi(t)$ decreases, the vertical arrow in the inset corresponds to the middle arrow in the main part of the figure, the introduction of even a small discrepancy into the condition of the synchronization of frequencies (13) transforms the frequency of rotation of Ω to zero (see the upper inset in the left-hand upper corner of the figure). The irrational value $\omega_2 = 2–$

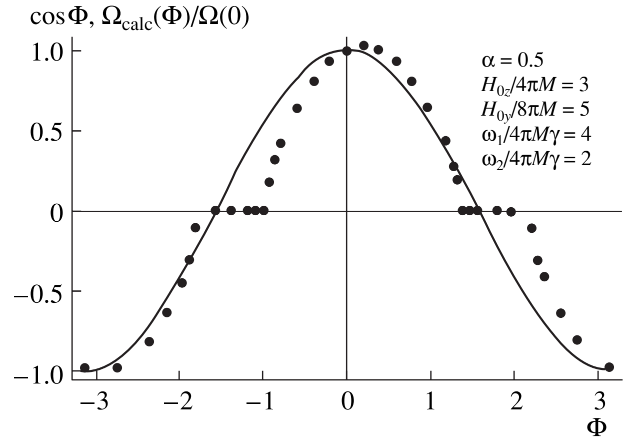

Fig. 2. Phase dependence of the frequency of rotation $\Omega(\Phi)$: solid curve, plotted using Eq. (17); points, numerical calculation via Eq. (2).

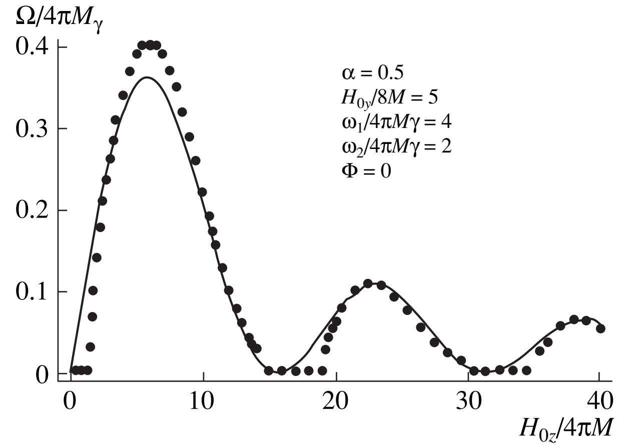

Fig. 3. Field dependence of the frequency of rotation $\Omega(H_{0z})$: solid curve, plotted using Eq. (17); points, numerical calculation via Eq. (2).

$2^{1/2}/10$, which is close to 2, was chosen in order to exclude the frequency-related synchronism possible at rational values of the ω_1/ω_2 ratio.

Returning back to the main curve, we note the existence of beatings with large periods $T_\Omega \sim 37$, which are marked by arrows; they cannot be described satisfactorily in the framework of the above approximations. Another circumstance referring to Fig. 1 is also related to the applicability of the perturbation theory. If we take that the formal parameter of smallness is $\varepsilon = \alpha H_{0y}[\omega_2(1 + \alpha^2)]$ (see Eq. (5.1)), then at the values of parameters indicated in Fig. 1 we have $\varepsilon = 1$. But since we consider the second-order effect, we should, as follows from Eqs. (10.2) and (12), additionally multiply ε^2 by $J_0(b)J_1(b)$. The maximum value of this multiplier is ~0.35, i.e., <1 ($b \sim 1$, so that we can speak of a qualitative applicability of (16) also at small values of $b = \gamma H_{0z}/[\omega_1(1 + \alpha^2)]$.

It is of interest to numerically estimate the possible contribution to Ω from processes that exceed the second order in which the theoretical formula (16) was obtained. Some of these processes should have a different phase dependence than that predicted by Eq. (16), i.e., $\sim\cos\Phi$. The character of the arising deviations can be obtained from Fig. 2 (the values of the parameters are given in the figure; they coincide with those given in Fig. 1). The solid line corresponds to the theoretical curve of $\cos\Phi$. Points were calculated by Eq. (2); they correspond to the values of the frequency of rotation $\Omega_{calc}(\Phi)/\Omega(0)$ depending on the phase $\Phi = \varphi_{0z} - 2\varphi_{0y}$ (for clearness, $\Omega_{calc}(\Phi)$ is normalized by $\Omega(0)$, the theoretical value of (16) at $\Phi = 0$. Although on the whole there is a satisfactory agreement between the calculated results and formula (16), in regions where $\cos\Phi \longrightarrow 0$, there is observed a characteristic discrepancy. In addition, the calculated picture is somewhat shifted to the right relative to the theoretical one.

Of large importance is the dependence of the frequency of rotation Ω on the amplitude of the external field H_{0z} oriented along the EA. The characteristic decaying oscillating dependence is displayed in Fig. 3 for the parameters taken from the set used in the preceding figures. Note the general agreement of numerical results with (16), except for those regions where the frequency Ω and the drift velocity linearly related to it are close to zero. The maximum value of the drift velocity, i.e., $|\langle \dot{q}(t)\rangle| = \Delta\Omega/\alpha$ for the data indicated in Fig. 3 is rather large: $\max(\Omega/4\pi\gamma M) \sim 1/2$ and $|\langle \dot{q}(t)\rangle| \approx 2\dot{q}_W$, where $\dot{q}_W = 2\pi\gamma M\Delta$ is the Walker velocity in materials with $Q > 1$ (usually, 1–10 m/s). From the experimental viewpoint, only moderate values of $H_{0z}/4\pi M$ are of importance at present.

According to (16), the frequency of rotation Ω (and related drift velocity) depend quadratically on the ac magnetic field $H_y(t)$ applied perpendicularly to the DW plane. To verify the quadratic law (and go beyond the limits of values of the damping α and the frequencies $\omega_{1,2}$ used in Figs. 1–3, Fig. 4 shows (discrete symbols) the results of calculations of $\Omega(H_{0y})$ based on the numerical solution of Eq. (2) for two values of the field amplitude $H_y(t)$ differing twofold.

The theoretical curve (17) for the field amplitude $H_{0y} = 5$ is plotted as a solid line. The data for $H_{0y} = 5$ calculated via Eq. (2) are plotted as square symbols; the data for $H_{0y} = 2.5$ (asterisks) are multiplied by a factor of 4 to provide their normalization with the above-mentioned theoretical curve $\Omega(H_{0y})$. A comparison of these data between themselves and with the theoretical curve shows that, with decreasing frequency and Gilbert damping factor α, we can speak of only a general qualitative agreement with the theory, which is especially well seen for the smaller value of H_{0y} ($H_{0y} = 2.5$). One of the possible explanations is that Eq. (16) obtained in

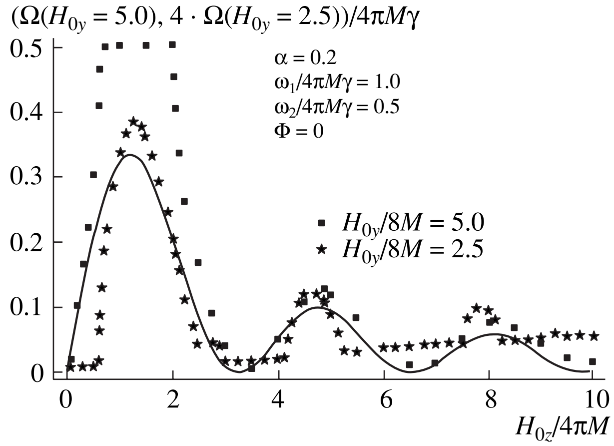

Fig. 4. Field dependence of the frequency of rotation $\Omega(H_{0z})$ at two values of the field H_{0y} perpendicular to the DW plane: solid curve, plotted using Eq. (17) at $H_{0y} = 5$; points, numerical calculation via Eq. (2); the data for $H_{0y} = 2.5$ are multiplied by 4.

the second order perturbation (see its derivation in Section 2) neglects the magnetostatic component. The role of magnetostatic effects is seen from the fact that, as is seen from Fig. 4, a better agreement with theory takes place when $H_{0y} = 5$, when, as can be supposed, the field $H_y(t)$ dominates the magnetostatic contribution proportional to $\sim\sin\psi\cos\psi$ in the right-hand side of Eq. (5.1). Therefore, it can be assumed that $H_{0y} = 2.5$ is insufficient to suppress magnetostatic effect, which contributes to (16) in higher-order perturbation theory and provides the inaccuracy of (16).

4. CONCLUSIONS

In conclusion, we note that in this work we suggest a two-frequency mechanism of excitation of domain walls (DWs). Its difference from the usually single-frequency mechanism is in that the two orthogonal components of the exciting field, one of which is collinear with the easy axis (EA), have different frequencies linked by a certain relationship. Such mechanism is sometimes more effective than a one-frequency one. Thus, for small amplitudes and frequencies of exciting fields according to Eq. (29) of [4] rewritten in variables (3) used in this paper we obtain the velocity of drift of a DW equal to $|\langle \dot{q}(t)\rangle| = H_{0x}H_{0z}(2Q\alpha)$, which, in contrast to (16), vanishes at $Q \longrightarrow \infty$. The corrections $\sim 1/Q$ to the DW drift which are proportional to $\sim 1/Q$ in the case of a strong field rotating in the basal plane, based on the numerical integration of the Landau–Lifshitz equations, were analyzed in [5], where also the effect of twisting on the DW drift was discussed for films with perpendicular anisotropy and $Q \gg 1$—a common object of experimental investigations; this effect, as can be expected, should become weaker with decreasing film thickness.

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