ISSN 0031-918X, The Physics of Metals and Metallography, 2008, Vol. 106, No. 6, pp. 543–547. © Pleiades Publishing, Ltd., 2008.
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THEORY OF METALS

Drift of a Twisted Domain Wall (TDW) in Perpendicularly Magnetized Films of Uniaxial Ferromagnetic Materials

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Received August 28, 2007

Abstract—Drift motion of a twisted domain wall (TDW) in a strong magnetic field circularly polarized in the plane of a perpendicularly magnetized film is considered. The twisted structure reduces the threshold for the regime of stationary motion of a TDW (on both the velocity of motion and frequency of the rotating field) in comparison with a one-dimensional DW of the Bloch or Néel type. The dependence of the boundary of optimum (from the experimental viewpoint) observation of drift of a TDW in the regime of stationary motion on the ratio of the width of a Bloch line to the thickness of the film has been determined.

PACS numbers: 75.70.Kw

DOI: 10.1134/S0031918X08120016

1. INTRODUCTION

Quite a common object of experimental investigations in the field of dynamics of domain walls (DWs) are perpendicularly magnetized films of uniaxial ferromagnetic materials (frequently, films of rare-earth iron garnets). Such films are characterized by the inequality $Q \equiv H_a/4\pi M > 1$, where Q is the quality factor; M is the magnetization; and H_a is the effective field of uniaxial anisotropy, which is perpendicular to the main surfaces of the film. The structure of 180° DWs in these films, due to the action of demagnetizing fields on the spins inside the DW, becomes inhomogeneous (two-dimensional) over the thickness of the film and is called "twisted." The dynamics of twisted DWs (TDWs) radically differs from the dynamics of one-dimensional (Bloch or Néel) DWs [1].

Passing to the drift of a DW, i.e., to its average translational motion in oscillating magnetic fields, note that the first theoretical works in this direction [2–4] (see also a review in [1]) were limited upon the calculation of the DW velocity only to a quadratic approximation in small amplitudes of external fields. The mechanism of drift in this approximation is considered as a consequence of the fact that under the conditions of forced precession caused by the action of oscillating magnetic fields the spins deep in the domains neighboring a 180° DW possess a different Zeeman energy, which causes a nonvanishing (upon the averaging over the time) internal pressure onto the plane of the DW and its translational displacement [1, 2]. Thus, the structure of the DW in this approximation makes no contribution to the drift velocity of the DW, at least, in the leading approximation. (For example, Schloemann [2] considered only small components of the induced magnetizations, which linearly depend on the exciting field through the tensor of the susceptibility of uniformly magnetized domains, and completely ignored spins inside the DW.)

A strongly nonlinear theory of drift for spatially one-dimensional DWs was constructed in [5, 6] by restricting the class of ferromagnets to materials with $Q > 1$. Below, we consider the case of a uniaxial ferromagnet circularly polarized in the basal plane in a strong magnetic field H ($8M < H < H_a$) at frequencies ω lower than the FMR frequency; for this case we revealed in [5] a mechanism of DW drift which differed from that considered in [2–4]. The drift appeared due to the entrainment of spins localized inside a DW by the rotating magnetic field; the contribution of spins in the domains has not been taken into account, since it is proportional to $\sim 1/Q$. It is understandable that within this approach the allowance for the internal structure of a DW in the perpendicularly magnetized films, i.e., the contribution of the twisted structure, becomes compulsory (an exception here can be only very thin films, in which the contribution from twisting is suppressed [7–12]).

In this work we extend the results of [5], which relate to the drift of a Bloch DW in a strong magnetic field rotating in the basal plane of a ferromagnet, to the case of a TDW. The primary attention is given to the determination of the frequency of the rotating field at which there occurs a violation of the stationary regime of drift. This frequency is linearly connected with a certain critical velocity of the TDW, which can be considered as an analog of the velocity at which the nucleation of a first horizontal Bloch line ("HBL nucleation") [1] known for the dynamics of TDWs in a dc magnetic field is observed and the passage into a nonstationary regime occurs.

2. EQUATIONS OF MOTION: NUMERICAL RESULTS

Let the origin of coordinates lie in the median plane of the film (of thickness h) and the $0z$ axis be perpendicular to its surfaces. Let the plane of the TDW be coincident with the plane $x0z$ and the magnetizations in the domains be $\mathbf{M}(y \longrightarrow \pm\infty, t) \longrightarrow \mp M\mathbf{e}z$. Since $Q > 1$, we can use, to an accuracy of $1/Q$, the Slonczewski equations instead of Landau–Lifshitz equations. The position of the center of the TDW $q(z, t)$ on the $0y$ axis and the azimuthal angle $\psi(z, t)$ of the vector of magnetization $\mathbf{M}(y - q(t), z, t)$ in the center of the DW satisfy the following equations and boundary conditions at the surfaces of the film:

$$\dot{\psi} + \alpha\dot{q} - H_z = \varepsilon^2 q''; \tag{1.1}$$

$$\dot{q} - \alpha\dot{\psi} = -\varepsilon^2\psi'' + (\sin\psi - H_z(z))\cos\psi + H_r\sin(\psi - \omega t); \tag{1.2}$$

$$\psi'(z = 1, t) = \psi'(z = -1, t) = q'(z = 1, t) = q'(z = -1, t) = 0. \tag{1.3}$$

Here, the dots and primes above the dependent variables mean the derivatives with respect to time t and coordinate z, respectively; the Zeeman contribution is given by the field $H_r > 0$, which rotates in the plane of film with a frequency ω; the first members in the right-hand side of Eqs. (1.1)–(1.2) are the contributions from the inhomogeneous exchange interaction; the second term in the right-hand side of (1.2) corresponds to the magnetostatic contributions (the first term in the parentheses is due to the local contribution; the second term, due to the contribution of "magnetic poles" on the surfaces of the film in the domains whose field $H_y(z) = \operatorname{arctanh}(z)$ leads to the twisting of the DW); α is the Gilbert damping parameter; $\varepsilon = 2\Lambda/h$, where $\Lambda = \sqrt{Q}\Delta$ is the width of the Bloch line (Δ is the width of the Bloch DW). Equations (1) are dimensionless: z is measured in units of $h/2$; t, in $1/(4\pi\gamma M)$ ($\gamma > 0$ is the magnetomechanical ratio); H_z, in $4\pi M$; H_r and $H_y(z)$, in $8M$; ω, in $4\pi\gamma M$; q, in Δ; and the TDW velocity $\dot{q}$, in $4\pi\gamma M\Delta$. Since the TDW dynamics was studied predominantly in a field H_z directed along the z axis, it is introduced into the set of Eqs. (1) for comparative purposes, where it is represented in units of $4\pi M$.

Before passing to an analysis of set (1), let us recall some results of the one-dimensional analysis (in (1) one should place $\varepsilon = 0$ and ignore the field $H_y(z)$) [5]. Let us introduce an angle $\Phi = \psi - \omega t$ between the vectors of the rotating field and the magnetization in the center of the TDW and use the condition $H_r > 1$, which makes it possible in the first approximation to ignore the term $\sin\psi\cos\psi$. In this approximation, set (1) reveals two regimes of motion: a stationary regime $\Phi = \Phi_0 = \text{const}$, $\dot{q} = \dot{q}_0 = \text{const}$ at the frequencies of the external field $\omega < \omega_c$, which yields

$$\dot{q}_0 = -\omega/\alpha, \quad \sin\Phi_0 = -(1+\alpha^2)\omega/\alpha H_r; \\ \omega_c = \alpha H_r/(1+\alpha^2), \quad |\dot{q}_0| \le |\dot{q}_c| = H_r/(1+\alpha^2); \tag{2}$$

and a precessional regime at $\omega > \omega_c$, where the average velocity of the DW (i.e., velocity of its drift) after reaching $|\dot{q}_c|$ falls monotonically with increasing frequency as $<\dot{q}(\omega \longrightarrow \infty)> \propto 1/\omega$. The influence of the magnetization oscillations caused by the effect of the member $\sin\psi\cos\psi$ on the stationary regime (2) at $H_r > 1$ is, according to [5], insignificant. The regime that is optimum for the experimental observation is, naturally, the first one.

Passing to the numerical integration of set (1) for the TDWs, note that the dynamics of TDWs also reveals two regimes of motion analogous to the above-considered ones. Below, we will examine only the stationary regime, when the motion of the TDWs can be represented in the form

$$\dot{q}(z, t) = \dot{q}_{00} + \delta\dot{q}(z, t), \quad \Phi(z, t) = \Phi_{00} + \delta\Phi(z, t), \tag{3}$$

where $\delta\dot{q}(z, t)$ and $\delta\Phi(z, t)$ are the inhomogeneous oscillations of the velocity and phase with a zero average near constant values $\dot{q}_{00}$ and Φ_{00}. However, now the stationary regime is limited to a frequency ω_n lying below ω_c (corresponding to the one-dimensional case): $\omega < \omega_n < \omega_c$ (see Eq. (2); the same refers also to the TDW velocity: $\dot{q}_n < \dot{q}_c$). The frequency ω_n (or the corresponding velocity $\dot{q}_n$) determines the threshold of nucleation of the first HBL in the TDW. In the case of thick films ($\varepsilon < 1$), there is formed in the TDW a well localized 360° HBL near one of the surfaces of the film; at $\varepsilon \sim 1$, the spatial disturbance of the structure along the coordinate z is less pronounced; no its sharp localization is observed. After the threshold of nucleation, motion of the 360° HBLs is observed over the entire thickness of the film and, depending on value of H_r, their multiple nucleation can occur. Note that in a dc field H_z there is observed a somewhat different picture [1]: at $\dot{q}_n$, near the surface of the film there appears an HBL of a small angle twist, which moves to the opposite surface and gradually becomes a ~360° HBL.

The numerical integration of set (1) in this work is limited to the determination of the $\omega_n(\varepsilon)$ dependence in the region of $0.1 < \varepsilon < 1.0$ for several values of the rotating field which lie in the region $1 < H_r < 5$. Figure 1 depicts several calculated curves of the $\omega_n(\varepsilon)$ dependences at various amplitudes of the rotating field H_r for the Gilbert damping parameter $\alpha = 0.4$. As follows from [7] (see also [9–12]), with increasing ε there occurs a suppression of twisting and passage to a spatially one-dimensional structure. In accordance with these results, the $\omega_n(\varepsilon)$ dependences represented in Fig. 1 grow on average with increasing ε and tend to their one-dimensional limits $\omega_c(H)$ (see (2)). However, this increase is not monotonic; the nature of deviations from the monotonicity depends on the amplitude H_r of the rotating magnetic field; with increasing H_r at ε = const, the frequency ω_n grows. A similar behavior of $\omega_n(\varepsilon)$ was revealed also at $\alpha = 0.2$ and 0.6.

3. THEORETICAL INTERPRETATION

Let us introduce into Eqs. (1) the previously mentioned dependent variable $\Phi = \psi - \omega t$ assuming that the motion takes on the form (3). Averaging of the linear equation (1.1) over the time and the film thickness (region $-1 < z < 1$), which is designated below by angular brackets $\langle\ldots\rangle$, shows that the average velocity of the TDW is connected with the frequency of the rotating field by the relationship (see (2))

$$\dot{q}_{00} = -\omega/\alpha, \qquad (4)$$

which is valid at $\omega < \omega_n$ irrespective of the approximations employed. The validity of Eq. (4) was confirmed by numerical calculations using various values of the parameters entering into (1), $\omega = \omega_n$ among them. (Note that if the TDW is advanced by the field H_z, the relationship (4) is replaced by $\dot{q}_{00} = H_z/\alpha$.) In turn, the averaging of (1.2) after elimination of $\dot{q}_{00}$ with the aid of (4) leads, instead of the appropriate formula in (2), to the expression

$$\omega = -\frac{\alpha}{1+\alpha^2}\langle H_y(z)\cos(\Phi + \omega t) + H_r \sin\Phi\rangle, \qquad (5)$$

where in the angular brackets we have the right-hand side of Eq. (1.2) in the approximation of $H_r > 1$ expressed through $\Phi(z, t)$ with allowance for the zeroing of the exchange term in view of the boundary conditions. In principle, Eq. (5) makes it possible to determine (from the known function $\Phi(z, t)$) the frequency of the nucleation as the maximum value of the right-hand side. In the limit of very large H_r, formula (5) coincides with (2) and, by the logic of derivation, the same coincidence takes place in the limit of very thin films $\varepsilon > 1$. The representation (5) makes it possible to make a qualitative conclusion that in the case of the smallness of oscillatory terms in (3) the frequency of nucleation ω_n should grow on average with increasing H_r or ε.

It follows from (5) that the explicit dependence of the $\omega_n(\varepsilon)$ curves on the Gilbert damping parameter α reduces to $\sim\alpha/(1 + \alpha^2)$. We recall that the same dependence is characteristic of the one-dimensional case when $\omega_c = H_r\alpha/(1 + \alpha^2)$ (see Eq. (2)). Figure 2 presents $\omega_n(\varepsilon)(1 + \alpha^2)/\alpha$ dependences calculated using solutions to Eqs. (1) at $H_r = 2$ for the cases of $\alpha = 0.2$, 0.4, and 0.6, which are grouped, as is seen from the figure, near a certain curve; this confirms the determining role of

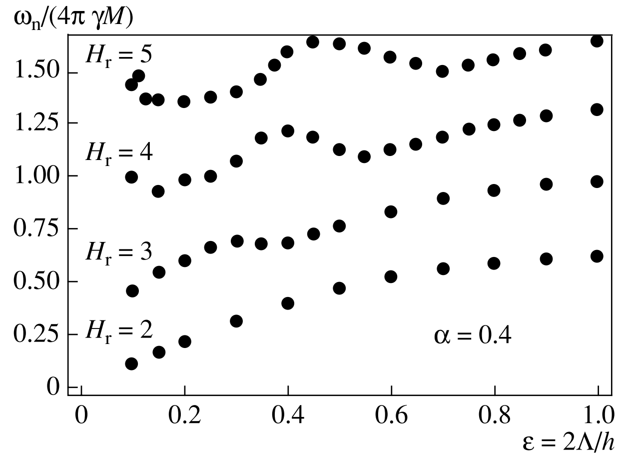

Fig. 1. Dependence of the frequency of nucleation ω_n of the first HBL in the structure of a TDW on the ratio of the width of the HBL to the thickness of the film for the values of the amplitudes of the magnetic field H_r, which rotates in the plane of the film, and on the Gilbert damping parameter α.

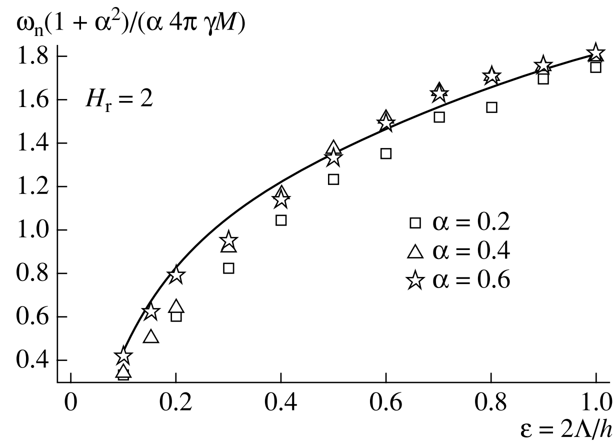

Fig. 2. Reduction of the calculated data on the frequency of nucleation ω_n to the universal curve for various values of the Gilbert damping parameter α.

the first factor in Eq. (5). However, such a behavior is observed only in the region of moderately large fields; as the field H_r increases, the $\omega_n(\varepsilon)$ dependences become nonmonotonic (see Fig. 1) and the data calculated for different α no longer lead to a common universal curve, as in the case of $H_r = 2$ in Fig. 2.

Let us try to explain the shape of the $\omega_n(\varepsilon)$ curve on the basis of the one-dimensional formula (2) for ω_c and known ideas concerning the rearrangement of the internal structure of TDWs in planar magnetic fields which occurs with the participation of HBLs. The static threshold for the appearance of 360° HBLs in the region of $z \approx 0$ of the TDW, according to [7, 8], is small in comparison with the amplitudes of the rotating field $H_r/8M > 1$, composing only $H/8M \sim a\varepsilon$ (a is a constant on the order of unity, and $\varepsilon \ll 1$). Therefore, this process cannot be responsible for the nucleation of HBLs at ω_n. Another process seems to be more acceptable, which occurs in higher fields perpendicular to the plane of the TDW through the nucleation of a HBL near one of the film surfaces [13].

In [13], the corresponding field of nucleation was estimated by the variational method to an accuracy of a "large logarithm" ~lnε, which proves to be insufficient for the purposes of this work and requires refinement. The correction to the energy functional corresponding to the right-hand side of Eq. (1.2) caused by the disturbance $\delta\psi(z)$ of the Néel configuration $\psi_0 = \pi/2$ has the form

$$\delta E = \int_{-1}^{1} dz\,\delta\psi(z)\left[-\varepsilon^2\frac{d^2}{dz^2} + H_r - 1 + \operatorname{arctanh}(z)\right]\delta\psi(z). \tag{6}$$

Assuming that the HBL nucleates near the lower surface of the film $z = -1$, we select the trial function in the previous form [13] as $\delta\psi = N\exp[-(1 + z)^2/(2b^2)]$, where b is the variational parameter. In the integral in Eq. (6) in view of the rapid decrease in $\delta\psi(z)$ we can enhance the upper limit to infinity and near $z_0 = -1$ use the expansion $\operatorname{arctanh}(z) = \frac{1}{2}\ln\frac{1+z}{2} + \frac{1+z}{4}\ldots$. For the region $-1 < z < \infty$, we have a normalizing factor $N = (2/(b\pi^{1/2}))^{1/2}$. The use of tabular integrals in (6) gives

$$\begin{aligned}\delta E &= \varepsilon^2/2b^2 + H_r - 1\\ &+ (-\gamma_E - 4\ln 2 + 2\ln b)/4 + b/(4\sqrt{\pi}),\end{aligned} \tag{7}$$

where $\gamma_E = 0.577\ldots$. The minimization of this expression with respect to b gives, with an accuracy to ε^2 : $b \approx 2^{1/2}\varepsilon(1 + \varepsilon/(4(2\pi)^{1/2}))$. After the substitution of the expression obtained into (7), we have from the condition $\delta E = 0$, omitting contributions proportional to $\sim\varepsilon^2$, the following expression:

$$H_{cr}(\varepsilon) = 1.41 - \frac{\ln\varepsilon}{2} - \frac{9\varepsilon}{16\sqrt{2\pi}}. \tag{8}$$

Now, we can try to extend the one-dimensional formula (2) for the critical frequency to the case of the nucleation of an HBL in the TDW as follows:

$$\omega_n(\varepsilon) = \frac{\alpha}{1+\alpha^2}(H_r - H_{eff}(\varepsilon)), \tag{9}$$

where the effective field of nucleation of an HBL is $H_{eff}(\varepsilon) = H_{cr}(\varepsilon) + \text{const}$. The constant in this expression is an adjustable parameter, which is necessary in view of the fact that expression (8) has been obtained in the purely static approximation, which assumes the perpendicularity of the external field to the plane of the CDW and ignores the delay of its structure from the rotating field, which occurs in the dynamic case. Note also that formula (9), which is semi-empirical by its nature, is valid only for the monotonic $\omega_n(\varepsilon)$ dependences such as the lower curve in Fig. 1. The curve that is obtained at const = –1, as can be seen from Fig. 2, satisfactorily describes the calculation data.

4. DISCUSSION OF RESULTS AND CONCLUSIONS

The mechanism of drift of a TDW considered in this work is in essence the same as for the one-dimensional DWs [5], i.e., the entrainment of spins localized inside TDWs by a magnetic field rotating in the plane of the film. However, the limitation of the stationary motion of the TDW with increasing frequency of rotation appears as a result of nucleation of 360° HBLs rather than the transition into the precessional regime, as in the one-dimensional case.

The basic results of the work are displayed in Fig. 1 in the form of numerically calculated dependences of the frequency ω_n, corresponding to the formation of the first HBL, on ε (the ratio of the HBL width to the film thickness, which is a critically important parameter in the theory of TDWs) at several values of the amplitudes of the rotating field and damping parameter. The frequencies of the nucleation of HBLs in a rotating field lie below the frequencies which limit the stationary regime in the one-dimensional case [5]; the transition to the one-dimensional behavior is observed in the region of $\varepsilon > 1$.

In the case of nonmonotonic $\omega_n(\varepsilon)$ dependences, the attempt of a theoretical interpretation encounters serious difficulties, since, as follows from the results of numerical calculations, multiple nucleation of HBLs occurs at the boundary of the stationary regime of motion. In the case of monotonic $\omega_n(\varepsilon)$ dependences (in the region of moderate values of the rotating-field amplitudes), it is shown in the work (see Fig. 2) that the basic $\omega_n(\varepsilon)$ dependence on the damping parameter coincides with that occurring in the one-dimensional case [5]. For the same range of the field amplitudes, a semi-empirical formula (9) has been suggested, which describes the $\omega_n(\varepsilon)$ dependence in the range of $0.1 < \varepsilon < 1.0$.

Let us present in conclusion some estimates of the parameters of drift of TDWs in an iron-garnet film with $4\pi M \sim 100$ G, $\gamma \sim 10^7$ rad/s, $\alpha \sim 0.2$, $\Delta \sim 5 \times 10^{-6}$ cm, $\varepsilon = 2\Lambda/h > 0.1$, and $H_r/8M = 2$. Under these conditions, $H_r \sim 120$ Oe and, according to (2), $\omega_c \sim 8 \times 10^8$ rad/s. It follows from Fig. 1 that ω_n, depending on ε, is equal to a few tenths of ω_c. The maximum velocity of the DWs is $|\dot{q}_c| = \omega_c\Delta/\alpha \sim 4 \times 10^4$ cm/s, so that for the TDWs the velocity $|\dot{q}_n|$ is on the order of several tenths of $|\dot{q}_c|$. It should, however, be taken into account that for the critical frequency (and velocity) there is also a limitation from below because of the presence of the field of the DW coercivity H_0, which in these materials is equal to or greater than 0.1 Oe. Under the action of the field H_z, which is perpendicular to the plane of the film, the motion of a TDW with a velocity $\dot{q} = \gamma\Delta H_z/\alpha$ occurs if $H_z > H_0$. Hence, the frequency of the rotating field is subject to a requirement that $\omega > \omega_0 = \gamma H_0$.

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