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SHORT COMMUNICATIONS

Exchange Reduction of the Magnetization Modulus in the Vicinity of a Bloch Point

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Abstract—The structure of the Bloch point (BP) core (magnetization "hedgehog") is determined taking into account the exchange-induced reduction of the length of the magnetization vector. The magnetization modulus vanishes at the center of a small sphere embracing the BP. The value of energy of such a BP in the vicinity of the Curie ferromagnetic point in the approximation of the second-order Landau phase transition is lower than the corresponding energy value calculated in the constant modulus approximation. The stability of the BP core to radial pulsations is demonstrated.

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It is well known that ferromagnetic samples may exhibit configurations of magnetization $\mathbf{M}(\mathbf{r})$ containing singularities of the form

$$\mathbf{m}(r) = \pm\mathbf{r}/r, \tag{1}$$

which are referred to as Bloch points (BPs) or magnetization "hedgehogs" [1, 2]. Here, $\mathbf{m}(r) = \mathbf{M}(\mathbf{r})/M_0$ is the magnetization normalized to its nominal value M_0. The literature devoted to BPs (mainly in magnetically uniaxial materials) is available and can be found in the above-cited monographs. In the case of cubic magnetic anisotropy, BPs were detected both at its positive sign (Fe) [3] and negative sign (YIG) [4]. If we consider modern applied aspects, the displacement of a BP along the magnetic vortex core is a natural way for magnetization reversal in permalloy nanodisks, although it requires strong constant magnetic fields [5, 6] (it should be noted that a more admissible practical method has been developed at present for this purpose; see, for example, [7]).

In estimating the energy of the core of a BP of type (1), the leading contribution comes from the exchange energy, while the remaining contributions taking into account the mechanisms of BP incorporation into a certain distribution of magnetization are considered as corrections [1, 2, 5]. According to estimates obtained in [1], the minimal value of the BP energy $E_{BP} \sim 0.5$ eV (0.8×10^{-12} erg) is quite large. Using the well-known expression (see [2, 5]) for the energy of BP (1),

$$E_{BP}^{(1)} = 8\pi A r \tag{2}$$

and truncating it at a distance $r = \Delta_B$ from the BP center, we obtain a still higher energy for moderate values of exchange hardness $A \sim 2 \times 10^{-7}$ erg/cm and the width $\Delta_B \sim 5 \times 10^{-6}$ cm of the Block domain wall (DW). These estimates, which are based on the assumption of constant magnetization magnitude $|\mathbf{m}(\mathbf{r})| = 1$, disagree with the mechanisms of thermal excitation of BPs, mentioned by some authors as a reason for the observed variation in the properties of domain structures [1].

Here, we pay attention to the fact that the observed reduction in the magnetization vector length (which is especially strong near the Curie point) may lead, along with the known considerable contribution of ultrarapid magnetization reversal to the process, to another significant effect, viz., a decrease in the BP energy as compared to the above estimates and to a decrease in its thermal nucleation threshold.

The exchange energy that makes the main contribution to the BP energy in the vicinity of the core has the form

$$E_{BP}^{(2)}(r) = \int[A(\nabla\mathbf{m})^2 + P(1-\mathbf{m}^2)^2]dxdydz = 4\pi P\Delta^3 f(r/\Delta), \tag{3.1}$$

$$f(r/\Delta) = \int_0^{r/\Delta}[m_r'(s)^2 + 2m_r^2(s)/s^2 + (1-m_r^2(s))^2]s^2 ds. \tag{3.2}$$

Here, A is the exchange hardness appearing in expression (2), $P = M_0^2/8\chi_p$ is the exchange energy density expressed in terms of the linear susceptibility χ_p of the ferromagnetic paraprocess (see [9]), and $\Delta = (A/P)^{1/2}$ is the length parameter determining the radius of the BP core (which generally differs from Δ_B).

Passage from the Cartesian coordinates used in formula (3.1) to spherical coordinates in (3.2) presumes that the sought solution is spherically symmetric, and only the radial component of the magnetization vector differs from zero: $m_r(r) = m(r)$, $m_\varphi = m_\theta = 0$. Isotropic exchange energy (3) is degenerate in spatial rotations of magnetization $\mathbf{m}(r) \longrightarrow \mathbf{R}(\theta_0, \varphi_0)\mathbf{m}(r)$, where θ_0 and φ_0 are constant angles and $\mathbf{R}$ is the rotation matrix. All solutions obtained using the above transformation from $m_r(r) = m(r)$ are equivalent from the energy point of view. Degeneracy in θ_0 and φ_0 is removed by additional interactions upon incorporation of a BP into the magnetic structure.

Relation (3.2) leads to the following equation for $m(r)$:

$$-(r^2m')'/r^2 + 2m/r^2 - 2m(1-m^2) = 0 \qquad (4.1)$$

with the boundary conditions $m(0) = 0$, $m(r \longrightarrow \infty) \longrightarrow 1$, where $r \longrightarrow r/\Delta$. Expression (4.1) defines two asymptotic forms satisfying the boundary conditions

$$m(r \longrightarrow 0) = Cr(1 - r^2/5 + \ldots), \quad m(r \longrightarrow \infty) = 1 - 2/r^2 + \ldots \qquad (5.1)$$

The numerical solution obtained using the method of shooting from the neighborhood of $r = 0$ is shown in Fig. 1 in comparison with asymptotic forms (5.1). It can be seen that the asymptotic forms successfully describe the entire domain of existence of $m(r)$. Parameter $C = 0.71565\ldots$ appearing in the first asymptotic form in (5.1) was determined numerically. The figure also shows for comparison the $m_\varphi(r)$ curve, viz., the circular component of the magnetization of a 2D vortex ($m_r = m_Z = 0$) with state index $S = 1$ [1] in cylindrical coordinates, which was calculated from the equation

$$-(rm_\varphi')'/r + m_\varphi/r^2 - 2m_\varphi(1-m_\varphi^2) = 0, \qquad (4.2)$$

with the boundary conditions

$$m_\varphi(r \longrightarrow 0) = C_1r(1 - r^2/4 + \ldots), \quad m_\varphi(r \longrightarrow \infty) = 1 - 1/(4r^2) + \ldots, \quad C_1 = 0.82475\ldots \qquad (5.2)$$

Passing to estimation of the BP energy, we use the asymptotic forms $f(r/\Delta \longrightarrow \infty) \longrightarrow 2r/\Delta$ to find the BP energy $E_{BP}^{(2)}(r/\Delta \longrightarrow \infty) = 8\pi Ar$, which coincides with quantity (2) obtained in the approximation $|\mathbf{m(r)}| = 1$. On the other hand, using the first asymptotic form in (5.1), we obtain $E_{BP}^{(2)}(r/\Delta \le 1) = 4\pi P(1 + 3C^2)r^3/3$ in the opposite limit. Since $dE_{BP}^{(2)}/dr > 0$ in accordance with relations (3), $E_{BP}^{(2)}$ has no extrema. Comparison of the two energy values using the relation

$$\kappa(r) = E_{BP}^{(2)}/E_{BP}^{(1)} \qquad (6)$$

shows (see Fig. 1) that $\kappa(r) < 1$. Thus, the reduction of the magnetization magnitude in the BP core is advantageous from the energy point of view, and the energy gain for $r/\Delta \sim 3$ can reach 50%.

Figure 1 shows that the main variation in $m(r)$ and $\kappa(r)$ takes place at distances $\sim \Delta = (A/P)^{1/2}$. It should be noted that the value of parameter Δ can be very small due to a large value of P (weakness of the para-

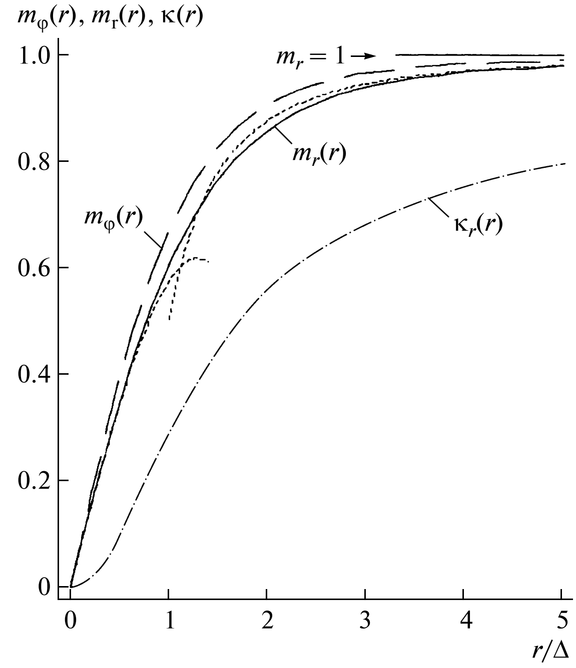

Fig. 1. Calculated dependence of the radial component $m_r(r)$ of BP magnetization on radius vector r (solid curve) together with asymptotic forms (5) (dotted curve); tangential magnetization $m_\varphi(r)$ of a 2D vortex (left dashed curve) is shown for comparison. The $\kappa(r)$ curve describes the ratio of the BP core energy (3) taking into account the reduction of the magnetization magnitude to the BP energy (2) disregarding this reduction (dot-and-dash curve).

process); in this case, the continual approach used here becomes inapplicable. The situation changes, however, in the vicinity of the Curie point. Energy density (3) is equivalent (to within a constant) to the free energy near the second-order Landau phase transition point in the ferromagnetic region,

$$F = \alpha(\nabla M)^2/2 - |a(T)|M^2 + bM^4, \qquad (7)$$

if we make the following substitutions in relations (3): $m = M/M_0$, $M_0(T) = (|a(T)|/2b)^{1/2}$, $P = bM_0^4$, and $A = \alpha M_0^2(T)/2$, where α is the square of the exchange length. With allowance for $a(T) = (1 - T/T_c)a_1$ ($a_1 > 0$), the exchange length parameter $\Delta = (\alpha/a_1)^{1/2}(1 - T/T_c)^{-1/2}$ increases significantly. Thus, the continual approach is justified at least in the temperature range below the Curie point, in which the classical Landau theory is valid. It should also be noted that in approximation (7), quantity Δ coincides with the width of the linear Zhirnov DW [10] (its existence is advantageous from the energy point of view in nanoconstrictions in spintronics [11]).

In conclusion, let us prove the stability of the solution to Eq. (4.1) for a BP relative to radial pulsations. For this purpose, we will use the relaxation equation (which holds near T_c) $\partial m/\partial t = -(\delta E_{BP}^{(2)}/\delta m)/(\tau M_0^2)$,

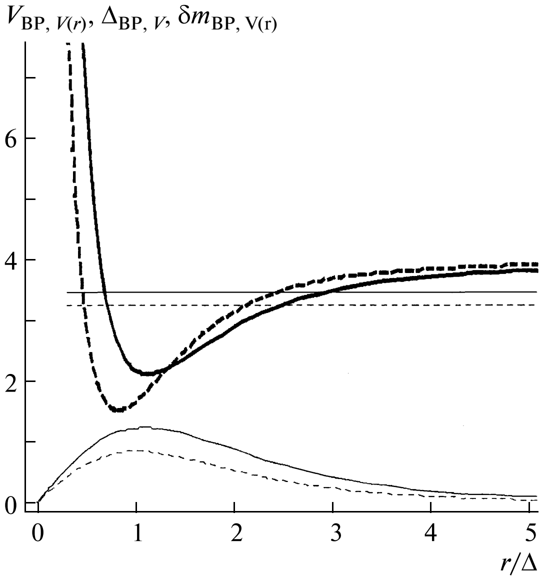

Fig. 2. Position of lower relaxation levels Ω_{BP} and Ω_V of magnetization oscillations in the cores of a BP and a 2D vortex (segments of horizontal straight lines) relative to the corresponding potentials V_{BP} and V_V in the linear stability equations (top); their eigenfunctions are plotted below. Solid and dashed curves correspond to BP and the 2D vortex, respectively.

where τ is the longitudinal relaxation time and $E_{BP}^{(2)}$ is the BP energy (3.2), which coincides with (4.1) in the static limit. For small radial perturbations $\delta m(r, t) = \exp(-\omega t)\delta m(r)$, we arrive at the linear stability equation

$$\Omega\delta m(r) = -(r^2\delta m')'/r^2 + V_{BP}(r)\delta m(r),$$
$$V_{BP}(r) = 2/r^2 + 6m_0^2(r) - 2, \quad (8)$$

where $m_0(r)$ denotes the solution to Eq. (4.1), $\Omega = \omega\tau M_0^2/2P$, and stability takes place when $\Omega > 0$. Numerical solution of Eq. (8) with boundary conditions $\delta m(0) = \delta m(r \longrightarrow \infty) = 0$ gives the lower attenuating relaxation level of radial BP pulsations $\Omega_{BP} = 3.4597$ ... and the corresponding eigenfunction $\delta m_{BP}(r)$, which are shown in Fig. 2 by solid curves.

For comparison, let us consider the stability equation for a two-dimensional vortex (4.2), which differs from Eq. (8) in the substitutions

$$(r^2\delta m')'/r^2 \longrightarrow (r\delta m')'/r,$$

$$V_{BP}(r) \longrightarrow V_V(r) = 1/r^2 + 6m_0^2(r) - 2,$$

in which $m_0(r) = m_\varphi(r)$. In this case, the lower level is $\Omega_V = 3.25379$ ... . The results for a cylindrical vortex are represented in Fig. 2 by dashed curves.

The main result of this study is the conclusion concerning a substantial decrease in the energy of the BP core in the vicinity of the Curie point (see Fig. 1), which facilitates thermal nucleation of the BP. Positive values of Ω_{BP} and Ω_V (see Fig. 2) reflect the stability of the core of a BP (and of a 2D vortex) to radial pulsations localized in the core region $r \sim \Delta$. Additional angular modes of BP oscillations depend on the magnetic structure into which a BP is incorporated [12, 13], and the stability of these modes is ensured by the stability of the external structure.

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